

**PROCESS INNOVATIONS AND REALLOCATION OF LABOUR:  
THE CASE IN WHICH A LABOUR UNION PROMOTES  
PRODUCTIVITY<sup>†</sup>**

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Abstract

In this paper, we consider the reallocation of workers when process innovation is different in each industry. We consider the condition which facilitates the reallocation of labour when there is process innovation in only one industry. When a limited number of workers utilize process innovation, the presence of a union might promote the reallocation of workers. This analysis suggests that process innovation in less-unionized sectors, such as information and network industries, spreads easily because of the influence of unions in other sectors.

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**I. INTRODUCTION**

In this paper, we consider the reallocation of workers when process innovation is different in each industry. In general, technological progress has a different influence in each industry and on each worker. A particular type of technological progress is likely to improve the productivity of only a particular type of worker who works in a specific industry. When such technological progress occurs where there are search-and-matching difficulties, the reallocation of workers differs from that predicted by standard models. This is because these models assume uniform productivity improvements.

In the long run, only industries and workers with high productivity will survive because these workers are trained to adjust to the new technology. In the short run, however, there exist workers and industries with both old and new technologies. In such cases, under what conditions does social productivity increase?

The effect of labour unions on unemployment is often considered. However, unions also influence the introduction of new technology. The standard textbook view is that union power lowers economic welfare (e.g., Blanchard 1997, Chapter 6). However, given that unions play a role in equalizing wages, they may also promote the reallocation of workers. They may have a similar influence under asymmetric information, when managers cannot observe the productivity levels of individual workers.

Acemoglu, Aghion and Violante (2001) analysed the relationship between skill-biased technical change, deunionization, and inequality. They emphasized the role of unions in reducing wage differences. They employed a two-sector model and showed that an increase in either productivity or training promotes deunionization. Acemoglu (1999) analysed the relationship between wage inequality and unemployment. In his model, there exist both separating and pooling equilibria concerning the firm's investment activities, and the numbers of each type of worker determines which equilibrium emerges. If the number or the productivity of skilled workers increases, the equilibrium shifts from pooling to separating and, therefore, inequality increases. Such a shift suggests the reallocation of labour. Referring to the results of these papers, we consider the implementation of process innovation under the assumption that a labour union is present.

In this paper, we consider the case in which two industries produce substitute goods. What condition facilitates the reallocation of labour when there is process innovation in only one industry? Reallocation depends not only on the extent of process innovation but also on the industry's share of employment and the proportion of workers who use the new technology, as well as on the presence of a union. When a

limited number of workers utilize process innovation, the presence of a union might promote the reallocation of workers when the union promotes wage equalization.

This paper is organized as follows. In Section II, we sketch the basic model. In Section III, we discuss the benchmark cases and use our model to derive some ‘standard’ union effects. In Section IV, we develop our model and apply it to cases in which a labour union is present in a part of the industry. We also determine the conditions under which productivity is increased. In Section V, we provide concluding remarks.

## II. THE MODEL

### II.1 The worker and the firm

Consider the case in which the productivity of workers and firms differs only in following process innovation.

We consider two types of worker, indexed by  $i = H, L$ ; the total number of workers is normalized to unity. Assume that the proportion of  $H$ -type workers is  $\phi$  and that the proportion of  $L$ -type workers is  $1 - \phi$ . Each worker expends one unit of labour, and can only discretely choose whether or not to work.

We consider two types of firms, indexed by  $i = A, B$ . Assume that the proportion of  $A$ -type firms is  $t$  and that the proportion of  $B$ -type firms is  $1 - t$ . The firm with vacancies participates in the labour market and hires unemployed workers. When the firm is employing workers, the worker’s business is expanding and a worker supplies the labour market with employment of his or her type.<sup>1</sup>

Without technological progress, the productivity levels of  $H$ - and  $L$ -type workers are equal;  $A$ - and  $B$ -type firms are also equally productive. Hence, firms and workers are matched at random. The proportion of  $H$ -type workers at firm  $A$  is  $t\phi$ , and  $(1-t)\phi$  workers of type  $L$  work at firm  $B$ . The proportion of workers of type  $H$  who work at firm  $B$  is  $(1-t)\phi$ , and  $(1-t)\phi$  workers of

type  $L$  work at firm  $B$ . The numbers of each type of worker in each firm are shown in Table 1.

TABLE 1 *The number of each type of worker in each firm*

	Firm $A$ ( $t$ )	Firm $B$ ( $1-t$ )
Worker $H$ ( $\phi$ )	$t\phi$	$(1-t)\phi$
Worker $L$ ( $1-\phi$ )	$t(1-\phi)$	$(1-t)(1-\phi)$

Suppose that only worker  $H$  achieves high productivity following process innovation. Moreover, if worker  $H$  works at firm  $A$ , then higher productivity is generated. Because the productivity of worker  $L$  is constant and does not depend on the type of firm, the output of worker  $L$  is normalized to unity. Therefore, the output of worker  $H$ , who works at firm  $A$ , is  $Y$ ; that of worker  $H$ , who works at firm  $B$ , is  $y$ ; and that of worker  $L$  is unity. (We assume that  $Y > y > 1$ .) For each matching, worker productivity is shown in Table 2.

TABLE 2 *The productivity of each matching*

	Firm $A$	Firm $B$
Worker $H$	$Y$	$y$
Worker $L$	1	1

Under these circumstances, the reallocation of workers is achieved through workers voluntarily changing jobs.

## II.2 Decision making

Let us consider a two-period model. Workers earn wages and firms gain profits

according to their matching output.

At the beginning of period 2, workers have two options: to continue working at the same firm, or to quit the firm and move on. Workers pursue strategies to maximize their payoffs. A worker who moves to a firm of the other type incurs a cost. The worker who opts to move and pay this cost can find a job at the type of firm to which he or she moves. The following expressions describe such a worker's circumstances:

$$V_{H,A} = w_{H,A} + \frac{1}{1+r} \max\{w_{H,A}, -c + w_{H,B}\} \quad (1)$$

$$V_{H,B} = w_{H,B} + \frac{1}{1+r} \max\{w_{H,B}, -c + w_{H,A}\} \quad (2)$$

$$V_{L,A} = w_{L,A} + \frac{1}{1+r} \max\{w_{L,A}, -c + w_{L,B}\} \quad (3)$$

$$V_{L,B} = w_{L,B} + \frac{1}{1+r} \max\{w_{L,B}, -c + w_{L,A}\} \quad (4)$$

In this context,  $V$  denotes the worker's value at the beginning of the period,  $w$  denotes the worker's wage, the first and second subscripts denote the type of worker and the type of firm in which the worker works, respectively, and  $r$  denotes the discount rate.

Next, we consider firms. A firm and a worker divide their output through Nash bargaining. We denote the worker's bargaining power as  $\alpha$ . Then, for example, the problem for firm  $A$ , which hires worker  $L$ , can be written as follows:

$$\max_{w_{H,A}} w_{H,A}^\alpha \times (Y - w_{H,A})^{1-\alpha} \quad (5)$$

Solving (5) yields the following conditions for the worker's wage:

$$w_{H,A} = \alpha Y \quad (6)$$

$$w_{H,B} = \alpha y \quad (7)$$

$$w_{L,A} = w_{L,B} = \alpha \quad (8)$$

Hence, workers' wages are proportional to their output levels.

### II.3 Average productivity

Because the productivity of type- $H$  workers differs between firms, total output is primarily determined by the allocation of type- $H$  workers. If all type- $H$  workers are employed at firm  $A$ , then total output and average productivity are maximized. The total maximum output is as follows:

$$Q_{max} = \phi Y + (1 - \phi) \quad (9)$$

By contrast, the allocation of type- $L$  workers does not affect average productivity. Hence, average productivity is maximized under the following situations:

- all type- $H$  workers work at firm  $A$ ; all type- $L$  workers work at firm  $B$ ;
- all type- $H$  and type- $L$  workers work at firm  $A$ ;
- all type- $H$  workers work at firm  $A$ ; type- $L$  workers work at any firm.

If the disutility of labour is constant, then welfare in each period is total output minus total cost.

### III. BENCHMARK CASES

To consider various configurations of union representation, we consider the case in which no union is present and the one in which a union is present in every firm.

#### III.1 No union

When there is no union to equalize wages in any firm, all workers receive wages that are proportional to their productivity levels.

In this case, only workers of type  $H$  at firm  $B$  have an incentive to change jobs. Workers of type  $L$  have no incentive to change jobs because their productivity is the same in any firm. Workers of type  $H$  who work at firm  $B$  change jobs when the following condition is satisfied:

$$-c + \alpha Y > \alpha y \quad (10)$$

$$\Leftrightarrow \alpha(Y - y) > c \quad (11)$$

That is, if the payoff from job change for worker  $H$  at firm  $B$  exceeds the cost of continuing to work at the firm, then the reallocation of labour occurs. Therefore, in the no-union case, (10) and (11) are necessary and sufficient conditions for average wage to be maximized.

In the absence of wage equalization by the union, efficiency is determined by dislocation costs and the extent of technological progress.

#### III.2 Unions in both firms

Next, we consider the case in which both firms have a union. In particular, we consider the wage-equalization role of the union.<sup>2</sup> In this case, the wages of workers at firm  $A$  (denoted by  $\ddot{w}$ ), and those at firm  $B$  (denoted by  $\bar{w}$ ), are set as follows:

$$\ddot{w} = \alpha(\phi Y + (1 - \phi)) \quad (12)$$

$$\bar{w} = \alpha(\phi y + (1 - \phi)) \quad (13)$$

Workers at firm  $A$  have no incentive to change jobs because  $\ddot{w}$  exceeds  $\bar{w}$ .

For workers at firm  $B$ , the gain from job change is  $\ddot{w}$ , and the cost is  $\bar{w} + c$ . Therefore, labour is reallocated if the following condition is satisfied:

$$\ddot{w} - \bar{w} > c \quad (14)$$

$$\text{where } \hat{w} - \bar{w} = \alpha\phi(Y - y) \quad (15)$$

For  $1 > \phi > 0$ , the condition for the reallocation of labour is strong, compared with the no-union case. This is one of the reasons why standard models imply that unions obstruct efficiency improvements.

#### IV. PARTIAL UNIONIZATION

In this section, we consider the case in which a union is present only in firm  $B$ , which derives a small benefit from technological progress. In period 1, firm  $B$  offers the union a wage of  $\bar{w}$ . At the beginning of period 2, worker  $H$  in firm  $B$  can change jobs to obtain a higher wage from firm  $A$ . Alternatively, worker  $L$  in firm  $A$  can change jobs to obtain the wage from firm  $B$ . The following four patterns are considered:

1. both  $H$  and  $L$  change their jobs;
2.  $H$  changes job, but  $L$  does not;
3.  $L$  changes job, but  $H$  does not;
4. neither  $H$  nor  $L$  change jobs.



In pattern 3, the union wage changes because the allocation of workers changes. The wage changes to  $\bar{w}'$ . In this case, because all type- $L$  workers work in firm  $B$ , the total number of type- $L$  workers in firm  $B$  is  $1 - \phi$ . The total number of type- $H$  workers remains at  $(1 - t)\phi$ . The proportion of type- $H$  workers in firm  $B$  changes to  $\frac{(1 - t)\phi}{(1 - \phi) + (1 - t)\phi}$ , and the proportion of type- $L$  workers becomes  $\frac{1 - \phi}{(1 - \phi) + (1 - t)\phi}$ . The new union wage is as follows.

$$\bar{w}' = \alpha \left( \frac{(1 - t)\phi}{1 - \phi t} y + \frac{1 - \phi}{1 - \phi t} \right) \quad (16)$$

In pattern 1, because only worker  $L$  is at firm  $B$ , the wage equals  $\alpha$ . In pattern 2, because only worker  $L$  is at firm  $B$ , the wage is also  $\alpha$ . The payoffs corresponding to workers' strategies are shown in Table 3.

TABLE 3 *The payoff of each matching*

		Worker $H$ 's strategy at firm $B$	
		Change	Do not change
Worker $L$ 's strategy at firm $A$	Change	$H : -c + \alpha Y,$ $L : -c + \alpha$	$H : \bar{w}',$ $L : -c + \bar{w}'$
	Do not change	$H : -c + \alpha Y,$ $L : \alpha$	$H : \bar{w},$ $L : \alpha$

#### IV.1 Nash equilibrium

In this subsection, we consider the conditions that need to be fulfilled for the workers' strategies to be the Nash equilibrium strategies.

**Case 1** ( $H$  : change;  $L$  : change) is the Nash equilibrium. For this case to occur, the following conditions need to be fulfilled.

$$H : -c + \alpha Y > \bar{w}' \quad (17)$$

$$L : -c + \alpha > \alpha \quad (18)$$

From the condition relating to  $L$ , it follows that  $0 > c$ . This violates the assumption. Thus, this case cannot be the Nash equilibrium.

**Case 2** ( $H$  : change;  $L$  : do not change) is the Nash equilibrium.

$$H : -c + \alpha Y > \bar{w} \quad (19)$$

$$L : \alpha > -c + \alpha \quad (20)$$

From the condition relating to  $H$ , the following inequality arises.

$$\phi < \frac{\alpha(Y-1) - c}{\alpha(y-1)} \equiv x_1 \quad (21)$$

The condition relating to  $L$  implies  $c > 0$ .

**Case 3** ( $H$  : do not change;  $L$  : change) is the Nash equilibrium.

$$H : \bar{w}' > -c + \alpha Y \quad (22)$$

$$L : -c + \bar{w}' > \alpha \quad (23)$$

From the condition relating to  $H$ , the following inequality arises.<sup>3</sup>

$$\phi > \frac{c - \alpha(Y - 1)}{tc + \alpha(1 - tY - (1 - t)y)} \equiv x_2 \quad (24)$$

From the condition relating to  $L$ ,

$$\phi > \frac{c}{tc + (1 - t)\alpha(y - 1)} \equiv x_3. \quad (25)$$

**Case 4** ( $H$ : do not change;  $L$ : do not change) is the Nash equilibrium.

$$H : \bar{w} > -c + \alpha Y \quad (26)$$

$$L : \alpha > -c + \bar{w}' \quad (27)$$

From the condition relating to  $H$ ,

$$\phi > x_1. \quad (28)$$

From the condition relating to  $L$ ,

$$\phi < x_3. \quad (29)$$

## IV.2 Properties of $x$

Let us delineate some properties of  $x$ , which characterize the equilibrium. First, we

identify the terminal values of  $x$ . For  $x_2$ , if  $t = 0$ :

$$x_2 \Big|_{t=0} = \frac{\alpha(Y-1) - c}{\alpha(y-1)} = x_1 \quad (30)$$

If  $t = 1$ :

$$x_2 \Big|_{t=1} = 1 \quad (31)$$

Thus,  $x_2$  crosses  $(\phi, t) = (1, 1)$ . For  $x_3$ , if  $t = 0$ :

$$x_3 \Big|_{t=0} = \frac{c}{\alpha(y-1)} \quad (32)$$

If  $t = 1$ :

$$x_3 \Big|_{t=1} = 1 \quad (33)$$

Thus,  $x_3$  also crosses  $(1, 1)$ .

Second, we identify conditions on the slopes of  $x$ . For  $x_2$ :

$$\text{sign} \frac{\partial x_2}{\partial t} = \text{sign}\{(\alpha(Y-1) - c)(c + \alpha(y-Y))\} \quad (34)$$

For  $x_3$  :

$$\text{sign} \frac{\partial x_3}{\partial t} = \text{sign}\{\alpha(y-1) - c\} \quad (35)$$

We consider the area  $t \in [0,1], \phi \in [0,1]$ . Therefore, these properties can be summarized as follows.

**Proposition 1.**

If  $\alpha(Y-1) > c > \alpha(Y-y)$  is satisfied, then  $x_2$  divides the area. If

$\alpha(y-1) > c$  is satisfied, then  $x_3$  divides the area.

Because  $x_2$  and  $x_3$  cross  $(1,1)$ , and because  $Y > y > 1$ , if these slopes are positive, then there exist areas in which the equilibrium is conditioned by these lines.

Which is higher,  $x_2$  or  $x_3$ ? Given that both lines cross  $(1,1)$ , we can answer this question by investigating the relationship in which  $t = 0$ .

**Proposition 2.**

If  $c > \alpha(Y-1)/2$  is satisfied, then  $x_3$  is located above  $x_2$ . If

$\alpha(Y-1)/2 > c$  is satisfied, then  $x_2$  is located above  $x_3$ .

These propositions show that several graphs are needed depending on technological progress and the level of cost, and on the relationships between them.

### IV.3 Numerical examples

To demonstrate using an example, we consider only the case in which  $Y-1 > y-1 > (Y-1)/2 > Y-y$ . In this case,  $Y$  is not so much larger than  $y$ . In this case,  $Y$  does not exceed  $y$ .<sup>4</sup> These situations are shown in Figures 1, 2, and 3. We explain the details below.

#### Case A:

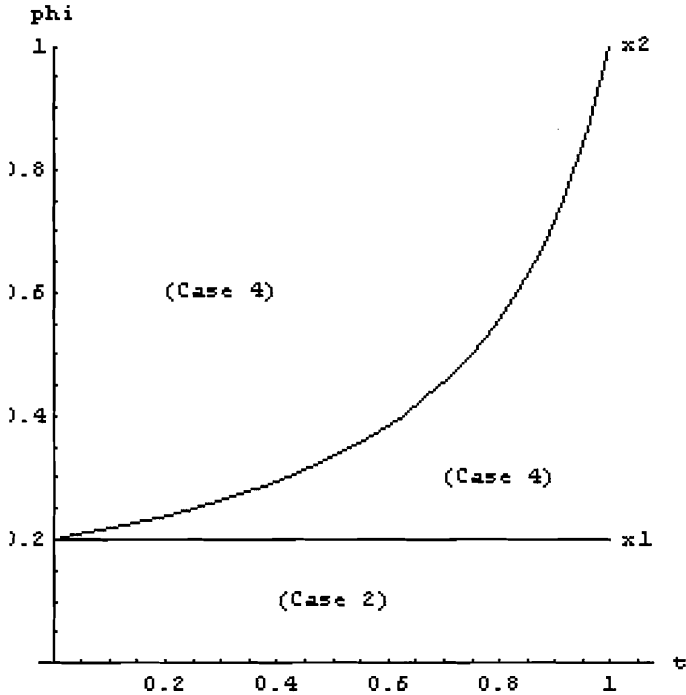
$c > \alpha(Y-1)$ . This case is trivial. The entire area ( $H$ : do not change;  $L$ : do not change) becomes the Nash equilibrium. Because the cost is too high, neither type of worker changes jobs.

#### Case B:

$\alpha(Y-1) > c > \alpha(y-1)$ . This is shown in Figure 1. The area below  $x_1$  ( $H$ : change;  $L$ : do not change) becomes the Nash equilibrium. When the proportion of type- $H$  workers is low, the union wage is low, and this is not profitable for worker  $L$  in firm  $A$ , but it is profitable for worker  $H$  in firm  $B$ . However, as  $\phi$  increases, the initial union wage is similar to the one obtained by worker  $H$  from firm  $A$ . Therefore, the area above  $x_1$  ( $H$ : do not change;  $L$ : do not change) becomes the Nash equilibrium.

Fig. 1. The relationship between  $x_1, x_2$ , and  $x_3$ , where

$$Y = 1.5, y = 1.4, \alpha = 0.5, c = 0.21.$$

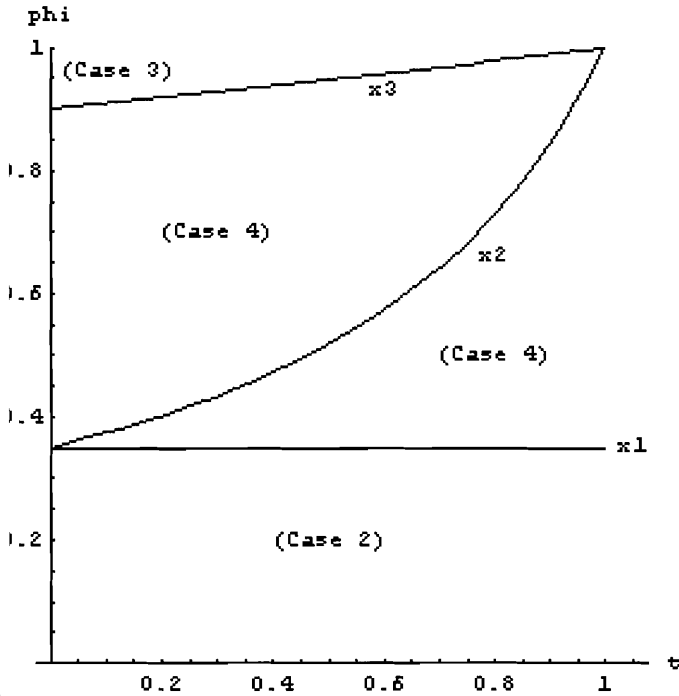


**Case C:**

$\alpha(y-1) > c > \alpha(Y-1)/2$ . This is shown in Figure 2. In this case, for a large  $\phi$  and a small  $t$  area, ( $H$ : do not change;  $L$ : change) becomes the Nash equilibrium. Because a large proportion of type- $H$  workers is at firm  $B$ , the union wage attracts worker  $L$  to firm  $A$ . For worker  $H$ , however, the union wage is similar to that paid by firm  $A$ .

Fig. 2. The relationship between  $x_1, x_2$ , and  $x_3$ , where

$$Y = 1.5, y = 1.4, \alpha = 0.5, c = 0.18.$$



**Case D:**

$\alpha(Y-1)/2 > c > \alpha(Y-y)$ . This is shown in Figure 3. This figure differs from Figure 2 in that there exists an area of mixed equilibrium. Denote the probability of worker  $H$  choosing the strategy of job change as  $p$ , and denote the probability of worker  $L$  doing so as  $q$ . The probability that the expected payoff is the same whatever the strategy chosen by worker  $H$  is as follows.



$$q = \frac{\alpha Y - \bar{w} - c}{\bar{w}' - \bar{w}} \quad (36)$$

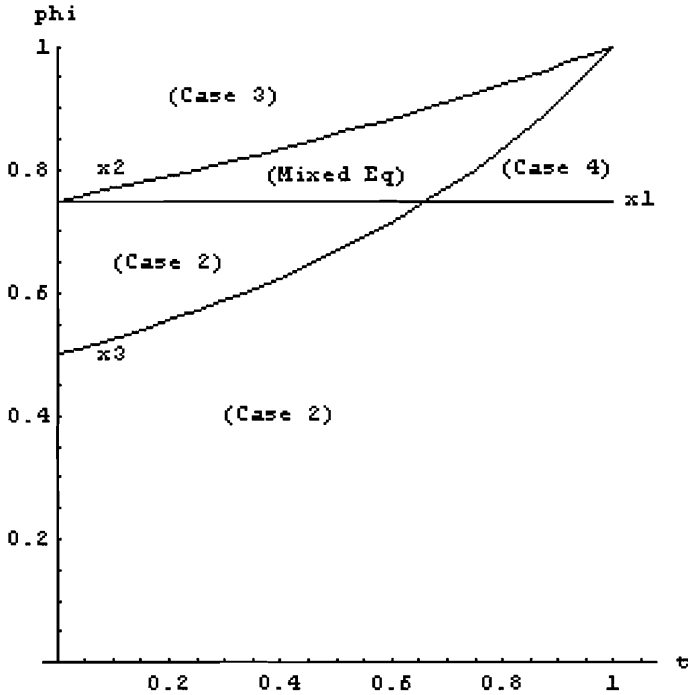
Similarly, the probability with regard to worker  $L$  is as follows.

$$p = \frac{\bar{w}' - \alpha - c}{\bar{w}' - \alpha} \quad (37)$$

In the area of mixed equilibrium, the combination that satisfies the above equations is the Nash equilibrium. This is an interesting case. Although the conditions remain the same, workers who change jobs coexist with those who remain in their jobs. This may explain why many workers do not act uniformly.<sup>5</sup>

Fig. 3. The relationship between  $x_1, x_2$ , and  $x_3$ , where

$$Y = 1.5, y = 1.4, \alpha = 0.5, c = 0.1.$$



**Case E:**

$\alpha(Y - y) > c$ . The entire area ( $H$ : change;  $L$ : do not change) becomes the Nash equilibrium. Because the cost is low, worker  $H$  changes jobs. Hence, in period 2, the union wage falls to unity. It is not profitable for worker  $L$  to change jobs; in fact, the worker incurs dislocation costs. Thus, worker  $L$  does not change jobs.

What is important about these figures is that  $x_1$  is important for worker  $H$  changing jobs. In the extreme situation, when  $\phi = 1$ , the parameter range is the same as that in the no-union case. At the opposite extreme, when  $\phi = 0$ ,  $x_1 > \phi$  satisfies  $\alpha(Y-1) > c$ . Hence, the range exceeds that in the no-union case. Because  $\bar{w}$  is monotonically increasing in  $\phi$ , when  $0 < \phi < 1$ , the range in which total output increases is greater when the industry is partly unionized. Therefore, partial unionization facilitates the reallocation of labour.

## V. CONCLUDING REMARKS

Our analysis suggests that under biased process innovation, wage equalization through the exercise of union power or asymmetric information can accelerate technological spread and efficiency improvement. This occurs in old-fashioned sectors with labour unions, which we analysed in Section IV.

This type of acceleration occurs because of wage equalization in an industry in which there is no process innovation, which drives out workers with potential. An industry in which there is process innovation may also drive out workers with potential. Therefore, unions in new industries may lower efficiency.

This analysis suggests that process innovation in less-unionized sectors, such as information and network industries, spreads easily because of the influence of unions in other sectors. By contrast, unions have a strong presence in the traditional smokestack industries. Consequently, process innovation in such an industry makes it hard to achieve average efficiency in the overall economy.

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<sup>1</sup> That is, in this model, a ‘firm’ can be interpreted as job opportunities.

<sup>2</sup> Such a modelling strategy has been used by, for example, Strand (2003). Garibaldi and Violante (2005) use such a strategy in the context of wage rigidity.

<sup>3</sup> The sign of this inequality is unambiguous when  $\alpha(Y-1) > c$ . This is because: (1) the

denominator of  $x_2$  is monotonically increasing in  $t$ ; (2) the denominator is

$\alpha(y-1) > 0$  when  $t = 0$ ; and (3) the denominator is  $\alpha(Y-1) - c$  when

$t = 1$ . Later, we show that  $x_2$  does not divide the area of our concern when this condition is not satisfied.

<sup>4</sup> By simple calculation, this is the case in which  $2y - Y > 1$ .

<sup>5</sup> Montgomery (1991) analyses such a mixed equilibrium. In his model, differences in firms’ valuations explain the differences in wages offered, varying queue length and the strategic complementarity of wages.

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