Prolegomena to Conditional Expected Utility Maximiser's Value Logic

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Abstract

In this paper we are concerned with value judgements on actions. The dominating rule for decision making under risk is [conditional] expected utility maximisation. Let us suppose the situation where an observer explains a conditional expected utility maximiser's value judgements on his actions. Then in general, the former does not always know the latter's degrees of belief and degrees of desire. The first problem now arises: how can an observer explain a conditional expected utility maximiser's value judgements on his actions, even when the former does not know the latter's degrees of belief and degrees of desire? (Explanation Problem) The first aim of this paper is to propose a new version of complete and decidable preference-based logic for goodness and badness based on Chisholm and Sosa's definition–conditional expected utility maximiser's value logic (CEUMVL) that can solve the Explanation Problem. In order to solve this problem, we resort to measurement theory. But generally preference-based logics are in danger of inviting the following problem: the development of a satisfactory logic of preference has turned out to be unexpectedly problematic. The evidence for this lies in the fact that almost every principle which has been proposed as fundamental to one preference logic has been rejected by another one. (Fundamental Problem of Intrinsic Preference) The second aim of this paper is to construct a model of CEUMVL that can avoid the Fundamental Problem of Intrinsic Preference.

Key Words: logic for goodness and badness, value judgement, conditional expected utility maximisation, measurement theory, representation theorem, projective geometry, filtration theory.

1 Introduction

In this paper we deal with value judgements that have the following linguistic form:

- It is good/bad/neutral + $\begin{cases} \text{that-clause.} \\ \text{to-infinitive.} \end{cases}$

Several authors have previously proposed various definitions of the *monadic* value predicate "good" and "bad" in terms of the *dyadic comparative* predicate "better". The best-known of these proposals is as follows:

- φ is good iff φ is better than $\neg\varphi$.
- φ is bad iff $\neg\varphi$ is better than φ.

This idea was proposed by Brogan ([3]) for the first time. Chisholm and Sosa ([4]) criticised this definition as follows. According to them, this definition is false, because there are states of affairs which are neither intrinsically good nor bad but which are nevertheless better than their negations. The following ([[4]: 245]) is an example of this.

Example 1 (Counterexample to Brogan's Idea by Chisholm and Sosa) *There are no unhappy egrets.* □

According to them, to rate a state of affairs intrinsically good, it should involve pleasure and not merely the absence of displeasure. Chisholm and Sosa proposed a different definition of "good" and "bad" in terms of "better" and provided a logic for "good" and "bad" based on this definition. When ψ is said to be neutral iff it is neither better nor worse than, that is, indifferent to $\neg\psi$, their definition is as follows:

- φ is good iff φ is better than some ψ which is neutral.
- φ is bad iff some ψ is which is neutral is better than φ.

Van Dalen ([29]) generalised Chisholm and Sosa's approach. His approach introduced a finite set of neutral propositions. Hansson ([6]) proposed a more comprehensive approach.

In this paper we are concerned with value judgements on *actions*. We can classify decision problems about actions into the following three types. We shall say that an agent is in the realm of alternative decision making under:

1. *Certainty* if each action is known to lead invariably to a specific outcome.

2. *Risk* if each action leads to one of a set of possible specific outcomes, each outcome occurring with a known probability. The probabilities are assumed to be known to the decision maker. ... Of course, certainty is a degenerate case of risk where the probabilities are 0 or 1.

3. *Uncertainty* if either action or both has as its consequence a set of possible specific outcomes, but where the probabilities of these outcomes are completely unknown or are not even meaningful. ([[13]: 13])

The dominating rule for decision making under risk is *[conditional] expected utility maximisation*. Neither [4], [29] nor [6] is concerned with value judgements on actions. On the other hand, in this paper we propose a logic for them. Let us suppose the situation where an *observer* explains a *conditional expected utility maximiser's* value judgements on his actions. Then in general, the former does not always know the latter's degrees of belief and degrees of desire. The first problem now arises:

Problem 1 (Explanation Problem) *How can an observer explain a conditional expected utility maximiser's value judgements on his actions, even when the former does not know the latter's degrees of belief and degrees of desire?* □

We call it the *Explanation Problem*. The *first* aim of this paper is to propose a new version of complete and decidable preference-based logic for goodness and badness based on Chisholm and Sosa's definition–*conditional expected utility maximiser's value logic* (CEUMVL) that can solve the Explanation Problem. The semantics of Packard's preference logic ([18]) is based on expected utility maximisation. But this logic is not complete. CEUMVL based on conditional expected utility maximisation, on the other hand, has the merit of being not only complete but also decidable. In order to solve this problem, we resort to *measurement theory*.[1] There are two fundamental problems with measurement theory:

1. the representation problem–justifying the assignment of numbers to objects or propositions,

2. the uniqueness problem–specifying the transformation up to which this assignment is unique.

[1] [20] gives a comprehensive survey of measurement theory. The mathematical foundation of measurement had not been studied before Hölder developed his axiomatisation for the measurement of mass ([7]). [12], [24] and [14] are seen as milestones in the history of measurement theory.

A solution to the former can be furnished by a *representation theorem*, which establishes that the chosen numerical system preserves the relations of the relational system. Representation theorems of [conditional] expected utility maximisation have the following form:

> If [and only if] an agent's preferences satisfy such-and-such conditions, there exist a probability function and a utility function such that he should act as a [conditional] expected utility maximiser.

There are at least two kinds of decision theory:

1. evidential decision theory,[2]
2. causal decision theory.[3]

The former is designed for decision makings that have statistical or evidential connections between actions and outcomes. The latter is designed for decision makings that have causal connections between actions and outcomes. Both theories adopt [conditional] expected utility maximisation as a main decision rule. Jeffrey ([9]) is a typical example of the former. Ramsey ([19]) is a typical example of the latter. Ramsey regarded degree of desire as attitude toward consequences but degree of belief as *propositional attitude*. Moreover, he regarded preference as attitude toward an ordered pair of gambles, that is, hybrid entities composed of consequences and propositions. In 1965 Jeffrey ([9]) developed an alternative to Ramsey's theory. He regarded both degree of desire and degree of belief as propositional attitudes. Moreover, he regarded preference as propositional attitude (attitude toward an ordered pair of propositions). In this sense we call Jeffrey's a *mono-set theory*. Its initial axiomatisation was provided in terms of measurement theory by Bolker ([2]) on the mathematics developed in [1]. Jeffrey ([8]) modified Bolker's axioms to accommodate null propositions. Domotor ([5]) axiomatised a *finite* version of mono-set theory. In mono-set measurement theories, Domotor's representation theorem is the only known one of conditional expected utility maximisation that has the "only if" part. Mono-set measurement theories are more suitable for the semantics of logic than non-mono-set ones like Savage's ([21]), for regarding propositions as the semantic values of sentences is simpler than regarding entities like acts (that is, functions from the set of possible worlds to the set of consequences) as those when we wish to provide logic with its semantics. So only by virtue of Domotor's representation theorem, an observer can explain, in a mono-set measurement theory,

[2] [9] gives a comprehensive survey of evidential decision theory.
[3] [11] gives a comprehensive survey of causal decision theory.

a conditional expected utility maximiser's inferences about his preferences in his actions, even when the former does not need to know the latter's degrees of belief and degrees of desire. In this paper we provide CEUMVL with a model based on Domotor's representation theorem, which enables CEUMVL to solve the Explanation Problem.

But generally preference-based logics are in danger of inviting the following problem. Von Wright ([30]) divided preferences into two categories: *extrinsic* and *intrinsic* preference. An agent is said to prefer φ_1 extrinsically to φ_2 if φ_1 is better than φ_2 in some explicit respect. So we can explain extrinsic preference from some explicit point of view. If we cannot explain preference from any explicit point of view, we call it intrinsic. Most preference logics that have been proposed are intrinsic but little attention has been paid to extrinsic preference. Von Wright ([31]) posed the following fundamental problem intrinsic preference logics faced.

Problem 2 (Fundamental Problem of Intrinsic Preference) *The development of a satisfactory logic of preference has turned out to be unexpectedly problematic. The evidence for this lies in the fact that almost every principle which has been proposed as fundamental to one preference logic has been rejected by another one.* □

We call it the *Fundamental Problem of Intrinsic Preference*. For example, the status of such logical properties as Transitivity, Contraposition, Conjunctive Expansion, Disjunctive Distribution and Conjunctive Distribution is as follows:

Example 2 (Variety of Preferences)

	von Wright ([30])	*Martin ([15])*	*Chisholm and Sosa ([4])*
Transitivity	+	+	+
Contraposition	−	+	−
Conjunctive Expansion	+	−	−
Disjunctive Distribution	−	−	−
Conjunctive Distribution	+	−	−

'+' denotes the property in question being provable in the logic in question. '−' denotes the property in question not being provable in the logic in question. Conjunctive Expansion says that an agent does not prefer φ_1 *to* φ_2 *iff he does not prefer* $\varphi_1 \& \neg\varphi_2$ *to* $\varphi_2 \& \neg\varphi_1$*. Disjunctive Distribution says that if he does not prefer* $\varphi_1 \vee \varphi_2$ *to* φ_3*, then he does not prefer* φ_1 *to* φ_3 *or does not prefer* φ_2 *to* φ_3*. Conjunctive Distribution says that if he does not prefer* φ_1 *to* φ_2 *and does not prefer* φ_3 *to* φ_2*, then he does not prefer* $\varphi_1 \vee \varphi_3$ *to* φ_2*.* □

The *second* aim of this paper is to construct a model of CEUMVL that can avoid the Fundamental Problem of Intrinsic Preference. According to Mullen ([16]), we can analyse its cause as follows. The adequacy criteria for intrinsic preference principles considered by preference logicians have been whether the principles are consistent with our *intuitions* of reasonableness. But each intuition often disagrees even on the fundamental properties. Different theories, such as ethics, welfare economics, consumer demand theory, game theory and decision theory make different demands upon the fundamental properties of preference. So if we would like to propose preference-based logic that can avoid the Fundamental Problem of Intrinsic Preference, it should be constructed not from intuition but from a *theory* or a *rule in a theory*, that is, it should be *extrinsic*. Conditional expected utility maximisation plays a central role in decision theory. When we provide CEUMVL with a model based on Domotor's representation theorem, we adopt conditional expected utility maximisation as a rule in decision theory that makes demands upon the fundamental properties of preference, which enables CEUMVL to avoid the Fundamental Problem of Intrinsic Preference.

The structure of this paper is as follows. In Section 2, we prepare the projective-geometric concepts for the measurement-theoretic settings, and define value preference space and value preference assignment, and state necessary and sufficient conditions for representation: Connectedness and Projectivity, and provide Domotor's representation theorem. In Section 3, we define the language $\mathcal{L}_{\mathsf{CEUMVL}}$ of CEUMVL, and define a Domotor-type structured Kripke model $\mathcal{M}$ for value preference, and provide CEUMVL with a truth definition, and provide CEUMVL with a proof system, and state logical properties of CEUMVL, and touch upon the proof of the soundness, completeness and decidability of CEUMVL.

2 Measurement-Theoretic Settings

The point of this section is as follows:

- We would like to sate necessary and sufficient conditions for *Domotor's representation theorem*.
- We can state them in terms of *exterior product*, *symmetric product* and *four-fold exterior product*.
- They all can be defined in terms of *four-fold Cartesian product*.

2.1 Projective-Geometric Concepts

We need some projective-geometric concepts to state Domotor's representation theorem. We define the preliminary concepts to the measurement-theoretic settings as follows:

Definition 1 (Preliminary Concepts) $\mathbf{W}$ *is a nonempty set of possible worlds. Let $\mathcal{F}$ denote a Boolean field of subsets of $\mathbf{W}$. We call $A \in \mathcal{F}$ a proposition.* $\square$

We define a characteristic function as follows:

Definition 2 (Characteristic Function I) *A characteristic function* $\widehat{\ }: \mathcal{F} \to \{0,1\}^{\mathbf{W}}$ *is one where for any $A \in \mathcal{F}$ we have $\widehat{A}: \mathbf{W} \to \{0,1\}$ such that*

$$\widehat{A}(w) := \begin{cases} 1 & \textit{if } w \in A, \\ 0 & \textit{otherwise,} \end{cases}$$

for any $w \in \mathbf{W}$. $\square$

Because it is impossible to characterise multiplication of probabilities and utilities in terms of union, intersection and preferences, we need a Cartesian product $\times$. $\widehat{\ }$ is defined also on Cartesian products of propositions:

Definition 3 (Characteristic Function II)

$$(A \times B)\widehat{\ }(w_1, w_2) := \begin{cases} 1 & \textit{if } w_1 \in A \textit{ and } w_2 \in B, \\ 0 & \textit{otherwise,} \end{cases}$$

for any $w_1, w_2 \in \mathbf{W}$. $\square$

By means of $\times$, we define an exterior product $\widehat{A} \wedge \widehat{B}$ as follows:

Definition 4 (Exterior Product) $\widehat{A} \wedge \widehat{B}$ *is a 3-valued random variable defined by*

$$\widehat{A} \wedge \widehat{B} := (A \times B)\widehat{\ } - (B \times A)\widehat{\ },$$

where '$-$' denotes subtraction. $\square$

Remark 1 *In short the exterior product is an anti-symmetric Cartesian product.* $\square$

Remark 2 *Intuitively,* $\widehat{A} \wedge \widehat{B}$ *can be measured by weighted utility differences* $P(A)P(B)(U(B) - U(A))$. □

Roughly speaking, projective geometry is represented in the language of *quadruples* of points. It suggests that we combine exterior products by means of a symmetric product $\odot$ as follows:

$$
\begin{aligned}
&(\widehat{A} \wedge \widehat{B}) \odot (\widehat{C} \wedge \widehat{D}) \\
&:= (\widehat{A} \wedge \widehat{B}) \wedge (\widehat{C} \wedge \widehat{D}) + (\widehat{C} \wedge \widehat{D}) \wedge (\widehat{A} \wedge \widehat{B}) = \\
&(A \times B \times C \times D)\widehat{\ } + (B \times A \times D \times C)\widehat{\ } + (C \times D \times A \times B)\widehat{\ } + (D \times C \times B \times A)\widehat{\ } \\
&-(A \times B \times D \times C)\widehat{\ } - (B \times A \times C \times D)\widehat{\ } - (C \times D \times B \times A)\widehat{\ } - (D \times C \times A \times B)\widehat{\ },
\end{aligned}
$$

where '+' denotes addition.

Remark 3 *Intuitively,* $(\widehat{A} \wedge \widehat{B}) \odot (\widehat{C} \wedge \widehat{D})$ *can be measured by weighted products of utility differences* $P(A)P(B)P(C)P(D)(U(B) - U(A))(U(D) - U(C))$. *Symmetric products can contribute to describing multiplication in [conditional] expected utility theory in terms of measurement theory.* □

By means of symmetric products, we define a four-fold exterior product $\triangle(\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D})$ as follows:

Definition 5 (Four-Fold Exterior Product) $\triangle(\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D})$ *is a 25-valued random variable defined by*

$$
\begin{aligned}
&\triangle(\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D}) := \\
&(\widehat{A} \wedge \widehat{B}) \odot (\widehat{C} \wedge \widehat{D}) + (\widehat{A} \wedge \widehat{C}) \odot (\widehat{D} \wedge \widehat{B}) + (\widehat{A} \wedge \widehat{D}) \odot (\widehat{B} \wedge \widehat{C}) = \\
&(A \times B \times C \times D)\widehat{\ } + (B \times A \times D \times C)\widehat{\ } + (C \times D \times A \times B)\widehat{\ } + (D \times C \times B \times A)\widehat{\ } \\
&-(A \times B \times D \times C)\widehat{\ } - (B \times A \times C \times D)\widehat{\ } - (C \times D \times B \times A)\widehat{\ } - (D \times C \times A \times B)\widehat{\ } \\
&+(A \times C \times D \times B)\widehat{\ } + (C \times A \times B \times D)\widehat{\ } + (D \times B \times A \times C)\widehat{\ } + (B \times D \times C \times A)\widehat{\ } \\
&-(A \times C \times B \times D)\widehat{\ } - (C \times A \times D \times B)\widehat{\ } - (D \times B \times C \times A)\widehat{\ } - (B \times D \times A \times C)\widehat{\ } \\
&+(A \times D \times B \times C)\widehat{\ } + (D \times A \times C \times B)\widehat{\ } + (B \times C \times A \times D)\widehat{\ } + (C \times B \times D \times A)\widehat{\ } \\
&-(A \times D \times C \times B)\widehat{\ } - (D \times A \times B \times C)\widehat{\ } - (B \times C \times D \times A)\widehat{\ } - (C \times B \times A \times D)\widehat{\ }.
\end{aligned}
$$

□

2.2 Value Preference Space and Value Preference Space Assignment

We define value preference space and value preference space assignment as follows:

Definition 6 (Value Preference Space and Value Preference Space Assignment)

- $\preceq_w$ *is a weak value preference relation on* $\mathcal{F} \times \mathcal{F}$.
- $A \preceq_w B$ *is interpreted to mean that* A *is not preferred in value to* B *in* w.
- $\sim_w$ *and* $\prec_w$ *are defined as follows:*
  - $A \sim_w B := A \preceq_w B$ *and* $B \preceq_w A$,
  - $A \prec_w B := A \preceq_w B$ *and* $A \not\sim_w B$.
- *For any* $w \in \mathbf{W}$, $(\mathbf{W}, \mathcal{F}, \preceq_w, \widehat{\ }, \times, +, -)$ *is called a value preference space.*
- *Let* **PS** *denote the set of all value preference spaces.*
- $\rho : \mathbf{W} \to \mathbf{PS}$ *is called a value preference space assignment.*

□

2.3 Conditions for Representation

We can state necessary and sufficient conditions for representation as follows:

1. $A \preceq_w B$ or $B \preceq_w A$ (***Connectedness***),
2. If ($A_i \preceq_w B_i$ and $C_i \preceq_w D_i$ for any $i < n$), then (if $A_n \preceq_w B_n$, then $D_n \preceq_w C_n$), where $\sum_{i \leq n} (\widehat{A_i} \wedge \widehat{B_i}) \odot (\widehat{C_i} \wedge \widehat{D_i}) = \triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n})$ (***Projectivity***).

Remark 4 *Projectivity essentially says that given an equality*

$$\sum_{i \leq n} P(A_i)P(B_i)P(C_i)P(D_i)(U(B_i) - U(A_i))(U(D_i) - U(C_i)) = 0,$$

the conditions $U(A_i) \leq U(B_i)$ *with* i *between 1 and* n *and* $U(C_i) \leq U(D_i)$ *with* i *between 1 and* $n-1$ *necessitate* $U(D_n) \leq U(C_n)$. *Zero on the right-hand side comes from the fact that the measure of* $\triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n})$ *happens to be equal to zero:*

$$P(A_n)P(B_n)P(C_n)P(D_n)((U(B_n) - U(A_n))(U(D_n) - U(C_n)) \\ +(U(C_n) - U(A_n))(U(B_n) - U(D_n)) + (U(D_n) - U(A_n))(U(C_n) - U(B_n))) = 0.$$

□

2.4 Domotor's Representation Theorem

We can prove Domotor's representation theorem as follows:

Theorem 1 (Representation) *When* $\mathbf{W}$ *is finite, for any* $w \in \mathbf{W}$*,* $(\mathbf{W}, \mathcal{F}, \preceq_w, \frown, \times, +, -)$ *satisfies Connectedness and Projectivity iff there are* $P_w : \mathcal{F} \to \mathbb{R}$ *and* $U_w : \mathcal{F} \backslash \emptyset \to \mathbb{R}$ *such that the following conditions hold for any* $A, B \in \mathcal{F} \backslash \emptyset$*:*

- $(\mathbf{W}, \mathcal{F}, P_w)$ *is a finitely additive probability space,*
- $A \preceq_w B$ *iff* $U_w(A) \leq U_w(B)$*,*
- *If* $A \cap B = \emptyset$*,* $U_w(A \cup B) = P_w(A|A \cup B)U_w(A) + P_w(B|A \cup B)U_w(B)$*,*
- *When* $A \in \mathcal{F}$*, if* $P_w(A) = 0$*, then* $A = \emptyset$*.*

□

Proof *Except that the proof is relative to world, it is similar to that of [[5]:184–194].* □

Remark 5 *In Theorem 1, we do not obtain the uniqueness result. But it does not matter when we provide* CEUMVL *with its model.* □

3 Conditional Expected Utility Maximiser's Value Logic CEUMVL

3.1 Language

The language $\mathcal{L}_{\mathsf{CEUMVL}}$ of CEUMVL is defined as follows:

Definition 7 (Language)

- *Let* $\mathbf{S}$ *denote a set of sentential variables,* $\{\nu_i\}_{i \in \mathbf{I}}$ *a finite indexed set of neutral sentences,* **WPR** *a weak value preference relation symbol, and* **FCP** *a four-fold Cartesian product symbol.* $\mathcal{L}_{\mathsf{CEUMVL}}$ *is given by the following rule:*

$$\varphi ::= s \mid \top \mid \nu_i \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \mathbf{WPR}(\varphi_1, \varphi_2) \mid \mathbf{FCP}(\varphi_1, \varphi_2, \varphi_3, \varphi_4),$$

where $s \in \mathbf{S}$*, and nestings of* **WPR** *do not occur.*

- $\bot, \vee, \to$ *and* $\leftrightarrow$ *are introduced by the standard definitions.*

- *We define a value indifference relation symbol* **IND** *and a strict value preference relation symbol* **SPR** *as follows:*

$$\mathbf{IND}(\varphi_1, \varphi_2) := \mathbf{WPR}(\varphi_1, \varphi_2)\&\mathbf{WPR}(\varphi_2, \varphi_1),$$
$$\mathbf{SPR}(\varphi_1, \varphi_2) := \mathbf{WPR}(\varphi_1, \varphi_2)\&\neg\mathbf{IND}(\varphi_1, \varphi_2).$$

- *We define a goodness relation symbol* **G***, a badness relation symbol* **B** *and a neutrality relation symbol* **N** *as follows:*

$$\mathbf{G}(\varphi) := \mathbf{SPR}(\nu_i, \varphi), \textit{for some } i \in \mathbf{I},$$
$$\mathbf{B}(\varphi) := \mathbf{SPR}(\varphi, \nu_i), \textit{for some } i \in \mathbf{I},$$
$$\mathbf{N}(\varphi) := \mathbf{IND}(\varphi, \neg\varphi).$$

- *The set of all well-formed formulae of* $\mathcal{L}_{\mathsf{CEUMVL}}$ *will be denoted by* $\Phi_{\mathcal{L}_{\mathsf{CEUMVL}}}$.

□

3.2 Semantics

3.2.1 DAG

In order to state $\sum_{i \leq n}(\widehat{A_i} \wedge \widehat{B_i}) \odot (\widehat{C_i} \wedge \widehat{D_i}) = \triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n})$ of Projectivity in logical terms, we use **FCP**. To provide **FCP** with a truth definition, we use a *directed acyclic graph (DAG)*. We got a hint about this idea from [17]. We define directedness as follows:

Definition 8 (Directedness) *A graph G is directed if G consists of a nonempty set* $\mathbf{W}$ *of vertices (possible worlds) and an irreflexive accessibility relation R on* $\mathbf{W}$. *G is denoted as* $(\mathbf{W}, R)$. □

We define a path as follows:

Definition 9 (Path) *A sequence $[w_1, \ldots, w_{n+1}]$ of vertices is a path of length n in G from w_1 to w_{n+1} if $(w_i, w_{i+1}) \in R$ for $i = 1, \ldots, n$.* □

By means of a path, we define a cycle.

Definition 10 (Cycle) *A cycle of length n is a path $[w_1, \ldots, w_n, w_1]$ from w_1 to w_1.* □

By means of a circle, we define acyclicity as follows:

Definition 11 (Acyclicity) *G is acyclic if G contains no cycles.* □

By means of directedness and acyclicity, we define a directed acyclic graph (DAG) as follows:

Definition 12 (DAG) *G is a directed acyclic graph (DAG) if G is both directed and acyclic.* □

We define some concepts:

Definition 13 (Parent, Child, Ancestor and Descendant) *w_1 is a parent of w_2 and w_2 is a child of w_1 if $(w_1, w_2) \in R$. w_1 is an ancestor of w_2 and w_2 is a descendant of w_1 if there is a path from w_1 to w_2.* □

Definition 14 (Ancestral Ordering) *$[w_1, \ldots, w_n]$ is an ancestral ordering of the vertices in $\mathbf{W}$ if for each $1 \leq i \leq n$ all the ancestors of w_i are ordered before w_i.* □

DAGs have the following important property.

Proposition 1 (Ancestral Ordering and DAG) *An ancestral ordering of the vertices in $\mathbf{W}$ exists iff G is a DAG.* □

3.2.2 Model

By means of a DAG, we define a Domotor-type structured Kripke model $\mathcal{M}$ for value preference as follows:

Definition 15 (Model)

- *$\mathcal{M}$ is a quintuple $(\mathbf{W}, R_{\mathbf{FCP}}, L, V, \rho)$, where:*
  - *$\mathbf{W}$ is a nonempty set of possible worlds.*
  - *$R_{\mathbf{FCP}}$ is an accessibility relation of $\mathbf{FCP}$ on $\mathbf{W} \times \mathbf{W}$.*
  - *$(\boldsymbol{W}, R_{\mathbf{FCP}})$ is a DAG.*
  - *$L : R_{\mathbf{FCP}} \rightarrow \{\pi_1, \pi_2, \pi_3, \pi_4\}$ is a function that assigns labels to the edges of the graph.*
  - *Any two edges leaving the same vertex have different labels.*
  - *Any vertex either has π_1-, π_2-, π_3- and π_4-labeled outgoing edges or none of them.*
  - *V is a truth assignment to each $s \in \mathbf{S}$ for each $w \in \mathbf{W}$.*

- ρ is a value preference space assignment that assigns to each $w \in \mathbf{W}$ $(\mathbf{W}, \mathcal{F}, \preceq_w, \widehat{\ }, \times, +, -)$, *where* $\mathcal{F}$ *includes a finite indexed set* $\{A_i : A_i \sim_w A_i^C\}_{i \in \mathbf{I}}$ *of neutral propositions, that satisfies Connectedness and Projectivity.*

- *For any* $s \in \mathbf{W}$, *by* $\pi_i(s)$ $(i = 1, 2, 3, 4)$ *we mean the unique* $t \in \mathbf{W}$ *such that* $R_{\mathbf{FCP}}(s, t)$ *and* $L(s, t) = \pi_i$ *if such world exists.*

□

3.2.3 Truth Definition

We can provide CEUMVL with the following truth definition:

Definition 16 (Truth) *The notion of* $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMVL}}}$ *being true at* $w \in W$ *in* $\mathcal{M}$, *in symbols* $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi$ *is inductively defined as follows:*

- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} s$ *iff* $V(w)(s) = \mathbf{true}$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \top$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \nu_i$ *iff* $[\![\nu_i]\!] \in \{A_i : A_i \sim_w A_i^C\}_{i \in \mathbf{I}}$ *and* $V(w)(\nu_i) = \mathbf{true}$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi_1 \& \varphi_2$ *iff* $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi_1$ *and* $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi_2$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \neg\varphi$ *iff* $(\mathcal{M}, w) \not\models_{\mathsf{CEUMVL}} \varphi$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \mathbf{FCP}(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ *iff* $(\mathcal{M}, \pi_1(w)) \models_{\mathsf{CEUMVL}} \varphi_1$ *and* $(\mathcal{M}, \pi_2(w)) \models_{\mathsf{CEUMVL}} \varphi_2$ *and* $(\mathcal{M}, \pi_3(w)) \models_{\mathsf{CEUMVL}} \varphi_3$ *and* $(\mathcal{M}, \pi_4(w)) \models_{\mathsf{CEUMVL}} \varphi_4$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \mathbf{WPR}(\varphi_1, \varphi_2)$ *iff* $[\![\varphi_1]\!]^{\mathcal{M}} \preceq_w [\![\varphi_2]\!]^{\mathcal{M}}$,

where $[\![\varphi]\!]^{\mathcal{M}} := \{w \in \mathbf{W} : (\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi\}$. *If* $(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \varphi$ *for all* $w \in \mathbf{W}$, *we write* $\mathcal{M} \models_{\mathsf{CEUMVL}} \varphi$ *and say that* φ *is valid in* $\mathcal{M}$. *If* φ *is valid in all Domotor-type structured Kripke models for value preference, we write* $\models_{\mathsf{CEUMVL}} \varphi$ *and say that* φ *is valid.* □

Remark 6 *Later we will provide the truth condition of a syntactic counterpart of Projectivity by means of Definition 17 and 18.* □

Remark 7 **FCP** *is a kind of modal operator. In* $\mathcal{M}$, *we have assumed that each possible world* w *has its proper four* $R_{\mathbf{FCP}}$*-accessible worlds* ($\pi_1(w), \pi_2(w), \pi_3(w)$ *and* $\pi_4(w)$) *or none of them, where* π_i *is defined by* $R_{\mathbf{FCP}}$ *and* L. *We have given the truth condition of* $\mathbf{FCP}(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ *in* w *in terms of the truth of* φ_1 *in* $\pi_1(w)$, *the truth of* φ_2 *in* $\pi_2(w)$, *the truth of* φ_3 *in* $\pi_3(w)$ *and the truth of* φ_4 *in* $\pi_4(w)$. □

Remark 8 *Because* $(\mathbf{W}, R_{\mathbf{FCP}})$ *is a DAG, by Proposition 1, an* **FCP**-*ancestral ordering of the vertices in* $\mathbf{W}$ *exists.* □

3.3 Syntax

3.3.1 Syntactic Counterpart of Projectivity

We devise a syntactic counterpart of Projectivity. By developing the idea of [23], we define a syntactic counterpart of Projectivity. Assume that

$$(3.3.1) \quad \sum_{i\leq n}(\widehat{A_i} \wedge \widehat{B_i}) \odot (\widehat{C_i} \wedge \widehat{D_i}) = \triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n}).$$

Then by means of Definition 5, we get

$$(3.3.2) \quad \sum_{i\leq n-1}(\widehat{A_i}\wedge\widehat{B_i})\odot(\widehat{C_i}\wedge\widehat{D_i})-(\widehat{A_n}\wedge\widehat{C_n})\odot(\widehat{D_n}\wedge\widehat{B_n})-(\widehat{A_n}\wedge\widehat{D_n})\odot(\widehat{B_n}\wedge\widehat{C_n}) = 0$$

Through transposition and simplification combined with the definition of $\odot$, we have

$$\begin{aligned}
&(3.3.3) \quad (A_1 \times B_1 \times C_1 \times D_1)\widehat{} + \cdots + (A_{n-1} \times B_{n-1} \times C_{n-1} \times D_{n-1})\widehat{} \\
&+(B_1 \times A_1 \times D_1 \times C_1)\widehat{} + \cdots + (B_{n-1} \times A_{n-1} \times D_{n-1} \times C_{n-1})\widehat{} \\
&+(C_i \times D_1 \times A_1 \times B_1)\widehat{} + \cdots + (C_{n-1} \times D_{n-1} \times A_{n-1} \times B_{n-1})\widehat{} \\
&+(D_1 \times C_1 \times B_1 \times A_1)\widehat{} + \cdots + (D_{n-1} \times C_{n-1} \times B_{n-1} \times A_{n-1})\widehat{} \\
&+(A_n \times C_n \times B_n \times D_n)\widehat{} + (C_n \times A_n \times D_n \times B_n)\widehat{} \\
&+(D_n \times B_n \times C_n \times A_n)\widehat{} + (B_n \times D_n \times A_n \times C_n)\widehat{} \\
&+(A_n \times D_n \times C_n \times B_n)\widehat{} + (D_n \times A_n \times B_n \times C_n)\widehat{} \\
&+(B_n \times C_n \times D_n \times A_n)\widehat{} + (C_n \times B_n \times A_n \times D_n)\widehat{} \\
&-(A_1 \times B_1 \times D_1 \times C_1)\widehat{} - \cdots - (A_{n-1} \times B_{n-1} \times D_{n-1} \times C_{n-1})\widehat{} \\
&-(B_1 \times A_1 \times C_1 \times D_1)\widehat{} - \cdots - (B_{n-1} \times A_{n-1} \times C_{n-1} \times D_{n-1})\widehat{} \\
&-(C_1 \times D_1 \times B_1 \times A_1)\widehat{} - \cdots - (C_{n-1} \times D_{n-1} \times B_{n-1} \times A_{n-1})\widehat{} \\
&-(D_1 \times C_1 \times A_1 \times B_1)\widehat{} - \cdots - (D_{n-1} \times C_{n-1} \times A_{n-1} \times B_{n-1})\widehat{} \\
&-(A_n \times C_n \times D_n \times B_n)\widehat{} - (C_n \times A_n \times B_n \times D_n)\widehat{} \\
&-(D_n \times B_n \times A_n \times C_n)\widehat{} - (B_n \times D_n \times C_n \times A_n)\widehat{} \\
&-(A_n \times D_n \times B_n \times C_n)\widehat{} - (D_n \times A_n \times C_n \times B_n)\widehat{} \\
&-(B_n \times C_n \times A_n \times D_n)\widehat{} - (C_n \times B_n \times D_n \times A_n)\widehat{} = 0.
\end{aligned}$$

For example, we can consider $\mathbf{FCP}(\varphi_1, \psi_1, \chi_1, \tau_1)$ to be a syntactic counterpart of $(A_1 \times B_1 \times C_1 \times D_1)\widehat{}$. So in terms of (3.3.3), we define $\mathbf{DC}_i$ (the disjunction of conjunctions of **FCP**s) that is the heart of a syntactic counterpart of Projectivity as follows:

Definition 17 (Disjunction of Conjunctions of FCPs) *For any* i $(0 \le i \le 4n+4)$, $\mathbf{DC}_i$ *is defined as the disjunction of all the following conjunctions:*

$$d_1\mathbf{FCP}(\varphi_1, \psi_1, \chi_1, \tau_1) \& \ldots \& d_{n-1}\mathbf{FCP}(\varphi_{n-1}, \psi_{n-1}, \chi_{n-1}, \tau_{n-1})$$
$$\& d_n\mathbf{FCP}(\psi_1, \varphi_1, \tau_1, \chi_1) \& \ldots \& d_{2n-2}\mathbf{FCP}(\psi_{n-1}, \varphi_{n-1}, \tau_{n-1}, \chi_{n-1})$$
$$\& d_{2n-1}\mathbf{FCP}(\chi_1, \tau_1, \varphi_1, \psi_1) \& \ldots \& d_{3n-3}\mathbf{FCP}(\chi_{n-1}, \tau_{n-1}, \varphi_{n-1}, \psi_{n-1})$$
$$\& d_{3n-2}\mathbf{FCP}(\tau_1, \chi_1, \psi_1, \varphi_1) \& \ldots \& d_{4n-4}\mathbf{FCP}(\tau_{n-1}, \chi_{n-1}, \psi_{n-1}, \varphi_{n-1})$$
$$\& d_{4n-3}\mathbf{FCP}(\varphi_n, \chi_n, \psi_n, \tau_n) \& d_{4n-2}\mathbf{FCP}(\chi_n, \varphi_n, \tau_n, \psi_n)$$
$$\& d_{4n-1}\mathbf{FCP}(\tau_n, \psi_n, \chi_n, \varphi_n) \& d_{4n}\mathbf{FCP}(\psi_n, \tau_n, \varphi_n, \chi_n)$$
$$\& d_{4n+1}\mathbf{FCP}(\varphi_n, \tau_n, \chi_n, \psi_n) \& d_{4n+2}\mathbf{FCP}(\tau_n, \varphi_n, \psi_n, \chi_n)$$
$$\& d_{4n+3}\mathbf{FCP}(\psi_n, \chi_n, \tau_n, \varphi_n) \& d_{4n+4}\mathbf{FCP}(\chi_n, \psi_n, \varphi_n, \tau_n)$$
$$\& e_1\mathbf{FCP}(\varphi_1, \psi_1, \tau_1, \chi_1) \& \ldots \& e_{n-1}\mathbf{FCP}(\varphi_{n-1}, \psi_{n-1}, \tau_{n-1}, \chi_{n-1})$$
$$\& e_n\mathbf{FCP}(\psi_1, \varphi_1, \chi_1, \tau_1) \& \ldots \& e_{2n-2}\mathbf{FCP}(\psi_{n-1}, \varphi_{n-1}, \chi_{n-1}, \tau_{n-1})$$
$$\& e_{2n-1}\mathbf{FCP}(\chi_1, \tau_1, \psi_1, \varphi_1) \& \ldots \& e_{3n-3}\mathbf{FCP}(\chi_{n-1}, \tau_{n-1}, \psi_{n-1}, \varphi_{n-1})$$
$$\& e_{3n-2}\mathbf{FCP}(\tau_1, \chi_1, \varphi_1, \psi_1) \& \ldots \& e_{4n-4}\mathbf{FCP}(\tau_{n-1}, \chi_{n-1}, \varphi_{n-1}, \psi_{n-1})$$
$$\& e_{4n-3}\mathbf{FCP}(\varphi_n, \chi_n, \tau_n, \psi_n) \& e_{4n-2}\mathbf{FCP}(\chi_n, \varphi_n, \psi_n, \tau_n)$$
$$\& e_{4n-1}\mathbf{FCP}(\tau_n, \psi_n, \varphi_n, \chi_n) \& e_{4n}\mathbf{FCP}(\psi_n, \tau_n, \chi_n, \varphi_n)$$
$$\& e_{4n+1}\mathbf{FCP}(\varphi_n, \tau_n, \psi_n, \chi_n) \& e_{4n+2}\mathbf{FCP}(\tau_n, \varphi_n, \chi_n, \psi_n)$$
$$\& e_{4n+3}\mathbf{FCP}(\psi_n, \chi_n, \varphi_n, \tau_n) \& e_{4n+4}\mathbf{FCP}(\chi_n, \psi_n, \tau_n, \varphi_n)$$

such that exactly i *of the* d_j*'s and* i *of the* e_j*'s are the empty string of symbols, the rest of them being the negation symbols.* □

By means of $\mathbf{DC}_i$, we define $\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i)$ (the disjunction of disjunctions of conjunctions of **FCP**s) that is a syntactic counterpart of Projectivity as follows:

Definition 18 (Disjunction of Disjunctions of Conjunctions of FCPs)

$$\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i) := \mathbf{DC}_1 \vee \ldots \vee \mathbf{DC}_{4n+4}.$$

□

By means of Definition 17 and 18, we can provide $\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i)$ with the following truth condition:

Proposition 2 (Truth Condition of $\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i)$)

$(\mathcal{M}, w) \models_{\mathsf{CEUMVL}} \mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i)$

iff

$(((\mathcal{M}, \pi_1(w)) \models_{\mathsf{CEUMVL}} \varphi_1$ *and* $(\mathcal{M}, \pi_2(w)) \models_{\mathsf{CEUMVL}} \psi_1$ *and* $(\mathcal{M}, \pi_3(w)) \models_{\mathsf{CEUMVL}} \chi_1$ *and* $(\mathcal{M}, \pi_4(w)) \models_{\mathsf{CEUMVL}} \tau_1)$ *and... and*
$((\mathcal{M}, \pi_1(w)) \models_{\mathsf{CEUMVL}} \chi_n$ *and* $(\mathcal{M}, \pi_2(w)) \models_{\mathsf{CEUMVL}} \psi_n$ *and* $(\mathcal{M}, \pi_3(w)) \models_{\mathsf{CEUMVL}} \tau_n$ *and* $(\mathcal{M}, \pi_4(w)) \models_{\mathsf{CEUMVL}} \varphi_n))$

or... or

$(((\mathcal{M}, \pi_1(w)) \not\models_{\mathsf{CEUMVL}} \varphi_1$ *or* $(\mathcal{M}, \pi_2(w)) \not\models_{\mathsf{CEUMVL}} \psi_1$ *or* $(\mathcal{M}, \pi_3(w)) \not\models_{\mathsf{CEUMVL}} \chi_1$ *or* $(\mathcal{M}, \pi_4(w)) \not\models_{\mathsf{CEUMVL}} \tau_1)$ *and... and*
$((\mathcal{M}, \pi_1(w)) \not\models_{\mathsf{CEUMVL}} \chi_n$ *or* $(\mathcal{M}, \pi_2(w)) \not\models_{\mathsf{CEUMVL}} \psi_n$ *or* $(\mathcal{M}, \pi_3(w)) \not\models_{\mathsf{CEUMVL}} \tau_n$ *or* $(\mathcal{M}, \pi_4(w)) \not\models_{\mathsf{CEUMVL}} \varphi_n))$. □

3.3.2 Proof System

We provide CEUMVL with the following proof system.

Definition 19 (Proof System)

- ***Axioms of*** CEUMVL

(*A1*) *All tautologies of classical sentential logic,*

(*A2*) $\mathbf{IND}(\nu_i, \neg\nu_i), i \in \mathbf{I}$ (***Neutrality***),

(*A3*) $\mathbf{WPR}(\varphi_1, \varphi_2) \vee \mathbf{WPR}(\varphi_2, \varphi_1)$
(***Syntactic Counterpart of Connectedness***),

(*A4*) $\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i) \rightarrow$
$((\mathbf{WPR}(\varphi_1, \psi_1) \& \mathbf{WPR}(\chi_1, \tau_1) \& \ldots \& \mathbf{WPR}(\varphi_{n-1}, \psi_{n-1}) \& \mathbf{WPR}(\chi_{n-1}, \tau_{n-1})) \rightarrow$
$(\mathbf{WPR}(\varphi_n, \psi_n) \rightarrow \mathbf{WPR}(\tau_n, \chi_n)))$
(***Syntactic Counterpart of Projectivity***),

(*A5*) $\mathbf{FCP}(\top, \top, \top, \top)$ (***Tautology and Four-Fold Cartesian Product***),

(*A6*) $\mathbf{FCP}(\varphi_1\&\varphi_2, \psi_1\&\psi_2, \chi_1\&\chi_2, \tau_1\&\tau_2)$
$\to (\mathbf{FCP}(\varphi_1, \psi_1, \chi_1, \tau_1)\&\mathbf{FCP}(\varphi_2, \psi_2, \chi_2, \tau_2))$
(***Conjunction and Four-Fold Cartesian Product 1***),

(*A7*) $(\mathbf{FCP}(\varphi_1, \mu, \nu, \xi)\&\mathbf{FCP}(\varphi_2, \mu, \nu, \xi)) \to \mathbf{FCP}(\varphi_1\&\varphi_2, \mu, \nu, \xi)$
(***Conjunction and Four-Fold Cartesian Product 2***),

(*A8*) $(\mathbf{FCP}(\lambda, \psi_1, \nu, \xi)\&\mathbf{FCP}(\lambda, \psi_2, \nu, \xi)) \to \mathbf{FCP}(\lambda, \psi_1\&\psi_2, \nu, \xi)$
(***Conjunction and Four-Fold Cartesian Product 3***),

(*A9*) $(\mathbf{FCP}(\lambda, \mu, \chi_1, \xi)\&\mathbf{FCP}(\lambda, \mu, \chi_2, \xi)) \to \mathbf{FCP}(\lambda, \mu, \chi_1\&\chi_2, \xi)$
(***Conjunction and Four-Fold Cartesian Product 4***),

(*A10*) $(\mathbf{FCP}(\lambda, \mu, \nu, \tau_1)\&\mathbf{FCP}(\lambda, \mu, \nu, \tau_2)) \to \mathbf{FCP}(\lambda, \mu, \nu, \tau_1\&\tau_2)$
(***Conjunction and Four-Fold Cartesian Product 5***),

(*A11*) $\neg\mathbf{FCP}(\varphi, \psi, \chi, \tau)$
$\leftrightarrow (\mathbf{FCP}(\neg\varphi, \psi, \chi, \tau) \vee \mathbf{FCP}(\varphi, \neg\psi, \chi, \tau)$
$\vee\mathbf{FCP}(\varphi, \psi, \neg\chi, \tau) \vee \mathbf{FCP}(\varphi, \psi, \chi, \neg\tau))$
(***Negation and Four-Fold Cartesian Product***).

- ***Inference Rules of*** CEUMVL

(*R1*) $\dfrac{\varphi_1 \quad \varphi_1 \to \varphi_2}{\varphi_2}$ (***Modus Ponens***),

(*R2*) $\dfrac{\varphi\&\psi\&\chi\&\tau}{\mathbf{FCP}(\varphi, \psi, \chi, \tau)}$ (***Four-Fold Cartesian Product Necessitation***),

(*R3*) $\dfrac{\varphi \leftrightarrow \psi \quad \chi' \textit{ is like } \chi \textit{ except for containing } \psi \textit{ in some place where } \chi \textit{ has } \varphi}{\chi \leftrightarrow \chi'}$
(***Replacement***).

A proof of $\varphi \in \Phi_{\mathsf{CEUMVL}}$ *is a finite sequence of* $\mathcal{L}_{\mathsf{CEUMVL}}$*-formulae having* φ *as the last formula such that either each formula is an instance of an axiom, or it can be obtained from formulae that appear earlier in the sequence by applying an inference rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{CEUMVL}} \varphi$*.* □

3.4 Logical Properties

The following logical properties are all provable in CEUMVL.

Proposition 3 (Logical Properties)

- $\vdash_{\mathsf{CEUMVL}} (\mathbf{G}(\varphi)\&\mathbf{WPR}(\varphi,\psi)) \to \mathbf{G}(\psi)$,
- $\vdash_{\mathsf{CEUMVL}} (\mathbf{B}(\varphi)\&\mathbf{WPR}(\psi,\varphi)) \to \mathbf{B}(\psi)$,
- $\vdash_{\mathsf{CEUMVL}} (\mathbf{G}(\varphi)\&\mathbf{B}(\psi)) \to \mathbf{SPR}(\psi,\varphi)$,
- $\vdash_{\mathsf{CEUMVL}} \mathbf{G}(\varphi) \to \neg\mathbf{B}(\varphi)$,
- $\vdash_{\mathsf{CEUMVL}} \mathbf{B}(\varphi) \to \neg\mathbf{G}(\varphi)$,
- $\vdash_{\mathsf{CEUMVL}} (\mathbf{SPR}(\varphi,\psi)\&\mathbf{N}(\varphi)) \to \mathbf{G}(\psi)$,
- $\vdash_{\mathsf{CEUMVL}} (\mathbf{SPR}(\varphi,\psi)\&\mathbf{N}(\psi)) \to \mathbf{B}(\varphi)$.

□

3.5 Metalogic

We can prove the soundness, completeness and decidability of CEUMVL. We cannot go into details because of limited space.

Theorem 2 (Soundness) *For any* $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMVL}}}$, *if* $\vdash_{\mathsf{CEUMVL}} \varphi$, *then* $\models_{\mathsf{CEUMVL}} \varphi$. □

Proof *The only difficulty of the proof of the soundness of* CEUMVL *is to show that all of instances of (A4) are true in all Domotor-type structured Kripke models for value preference. See [[28]: 15–17] about the details of the proof.* □

Theorem 3 (Completeness) *For any* $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMVL}}}$, *if* $\models_{\mathsf{CEUMVL}} \varphi$, *then* $\vdash_{\mathsf{CEUMVL}} \varphi$. □

Proof *We prove the completeness of* CEUMVL *by developing the idea of Segerberg ([23]) that we modify filtration theory in such a way that completeness can be established by a representation theorem in measurement theory. See [[28]: 17–22] about the details of the proof.* □

Theorem 4 (Decidability) CEUMVL *is decidable.* □

Proof *We prove the decidability of* CEUMVL *in terms of the finite model property that every non-theorem of* CEUMVL *fails in a Domotor-type structured Kripke model for value preference with only finitely many elements. See [[28]: 22] about the details of the proof.* □

4 Conclusion and Further Investigation

In this paper we have proposed a new version of complete and decidable extrinsic preference-based logic for goodness and badness based on Chisholm and Sosa's definition–conditional expected utility maximiser's value logic (CEUMVL) that can solve the Explanation Problem and avoid the Fundamental Problem of Intrinsic Preference by means of Domotor's representation theorem.

This paper is only a part of a larger measurement-theoretic study. We are now trying to construct such logics as *dyadic deontic logic* ([27]), *threshold utility maximiser's preference logic* and *logic of questions and answers* by means of measurement theory.

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