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Lags in Policy Response and Macroeconomic Stability *

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Ⅰ. Introduction

One of the influential objections to Keynesian Stabilization policy rests on the 'lags in policy response'. For example, a long time ago Friedman (1948) asserted that the government's stabilization policy may contribute to accelerate rather than suppress the instability of the capitalist economy because of the existence of the policy lag. However, as far as we acknowledge, even now there exist few fromal analyses of the effects of the policy lag on macroeconomic stability except Phillips' pioneering works (1954, 1957)(1).

The purpose of the present paper is to construct a formal model which is designed to analyze the effects of the lags in policy response on macroeconomic stability. In the next section, we formulate a very simple model of macroeconomic instability which is based on a dynamic version of Keynesian IS-LM system, and in section three, we introduce the policy lag into the model to investigate the relationships between the dynamic stability of the system and the delay of the policy response. We shall also present the results of comparative static and dynamic analyses with respect to some structural parameters.

II. A Simple Model of Macroeconomic Instability

〈Symbols〉

Y=gross real national income ; C=real private consumption expenditure ; I=gross real private investment expenditure ; G= real government expenditure ; T=real income tax ; M=nominal money supply ; H=nominal high-powered money ; $m \equiv M/H$=money multiplier ; r=nominal rate of interest ; p=price level.

〈System of Equations〉

Our model of macroeconomic instability, which is basad on a dynamic version of simple IS-LM system, consists of the following system of equations(2)(3).

$$\dot{Y}(t)=\alpha[C(t)+I(t)+G(t)-Y(t)]\ ;\ \alpha>0 \tag{1}$$

$$C(t)=c(Y(t)-T(t))+C_0\ ;\ 0<c<1,\ C_0\geqq 0 \tag{2}$$

$$I(t)=I(Y(t),\ r(t))\ ;\ I_Y\equiv\partial I(t)/\partial Y(t)>0,\ I_r\equiv\partial I(t)/\partial r(t)<0 \tag{3}$$

$$T(t)=\tau Y(t)-T_0\ ;\ 0<\tau<1,\ T_0\geqq 0 \tag{4}$$

$$G(t)=\bar{G}\equiv\text{constant.}\geqq 0 \tag{5}$$

$$H(t)=\bar{H}\equiv\text{constant.}>0 \tag{6}$$

$$M(t)/p(t)\equiv m(r(t))H/p(t)=L(Y(t),\ r(t))$$

$$; \; m \geqq 1, \quad m_r \equiv m'(r(t)) \geqq 0, \quad L_Y \equiv \partial L/\partial Y(t) > 0,$$

$$L_r \equiv \partial L/\partial r(t) < 0 \tag{7}$$

$$p(t) = p(Y(t)) \; ; \; p_Y \equiv p'(Y(t)) \geqq 0 \tag{8}$$

Eq. (1) implies that the real output fluctuates according to the excess demand in the goods market. Equations (2) through (4) are the consumption function, the investment function and the income tax function respectively. Eq. (7) is the equilibrium condition in the money market. Eq. (8) is the aggregate supply function.

<Analysis of the solution>

Substituting equations (2) through (5) into Eq. (1), we have the following equation.

$$\dot{Y}(t) = \alpha[I(Y(t), \; r(t)) - \{1 - c(1-\tau)\}Y(t) + C_0 + \bar{G} + cT_0] \tag{9}$$

Substituting equations (6) and (8) into Eq. (7), we also have

$$m(r(t))\bar{H}/p(Y(t)) = L(Y(t), \; r(t)). \tag{10}$$

Solving Eq. (10) with respect to r (t) gives

$$r(t) = r(Y(t)) \; ; \; r_Y \equiv r'(Y(t))$$

$$= -\{\underset{(+\text{ or }0)}{(m\bar{H}p_Y/p^2)} + \underset{(+)}{L_Y}\}/\{\underset{(-)}{L_r} - \underset{(+\text{ or }0)}{(m_r\bar{H}/p)}\} > 0 \tag{11}$$

; which is nothing but the 'LM equation'.

Substituting Eq. (11) into Eq. (9), we obtain the following simple differential equation, which is the fundamental equation in our system.

$$\dot{Y}(t) = \alpha[I(Y(t), \; r(Y(t))) - \{1 - c(1-\tau)\}Y(t) + C_0 + \bar{G} + cT_0]$$

$$\equiv f(Y(t)) \tag{12}$$

Let us *assume* that Eq. (12) has a unique stationary solution $Y^* > 0$. We shall refer to Y^* as the 'equilibrium national income'.

Differentiating Eq. (12) gives

$$d\dot{Y}(t)/dY(t)\Big|_{Y=Y^*} \equiv f'(Y^*) = \alpha[(\underset{(+)}{I_Y^*}+\underset{(-)}{I_r^*}\underset{(+)}{r_Y^*})-\underbrace{\{1-c(1-\tau)\}}_{(+)}] \tag{13}$$

where the asterisk (*) implies that each value is evaluated at the equilibrium point.

It is clear from Eq. (13) that the equilibrium point is locally stable if $I_Y^*+I_r^*\ r_Y^* < 1-c(1-\tau)$, while it is locally unstable if $I_Y^*+I_r^*\ r_Y^* > 1-c(1-\tau)$. Now, we shall assume that

Assumption 1. $\underset{(+)}{I_Y^*}+\underset{(-)}{I_r^*}\ \underset{(+)}{r_r^*} > 1-c(1-\tau)$.

Under this assumption, the destabilizing effect on the entrepreneur's investment activities (I_Y^*) outweighs the stabilizing effect through the money market ($I_r^*\ r_Y^*$), so that the system becomes unstable. For example, the rise of national income induces the rise of the investment demand, which induces further rise of national income through the growth of the effective demand. Of course, this process is somewhat mitigated through the depressing effect of the rise of the rate of interest on the investment demand, but, this effect is relatively weak under *Assumption 1*. Therefore, this process of the cumulative disequilibrium will persist until the changes of the capital stock together with the nonlinearity of the system defend the system from infinite divergency(4).

Now, let us suppose that the government carries out the stabilization policy by changing the government's expenditure in the following way *without time lag*.

$$G(t)=G_0+\beta(\bar{Y}-Y(t))\ ;\ \beta>0,\ \bar{Y}\equiv\text{constant.}\geqq 0 \tag{14}$$

If we replace Eq. (5) with Eq. (14), the fundamental dynamical equation (Eq. (12)) is modified as

$$\dot{Y}(t)=\alpha[I(Y(t),\ r(t))-\{1-c(1-\tau)+\beta\}Y(t)+C_0+G_0+\beta\bar{Y}+cT_0]\equiv g(Y(t);\ \beta). \qquad (15)$$

The local stability condition of this system is given as follows.

$$d\dot{Y}(t)/dY(t)\Big|_{Y=Y^*}\equiv g'(Y^*)=\alpha[I_Y^*+I_r^*r_Y^*-\{1-c(1-\tau)+\beta\}]<0 \qquad (16)$$

Therefore, the system becomes locally stable if $\beta>I_Y^*+I_r^*r_Y^*-\{1-c(1-\tau)\}\equiv a>0$.

Ⅲ. Lags in Policy Response and Dynamic Stability of the System

In the previous section, we have shown that the sufficiently 'activistic' stabilization policy can stabilize the economic system if the government's expenditure can respond to the changes of national income *instantaneously*. However, as Friedman (1948) pointed out, the government's response is apt to lag behind the changes of national income. In this section, we shall explicitly introduce the time lag in policy response into the model.

Now, let us suppose that the goverment's expenditure is determined by the following rule.

$$G(t)=G_0+\beta(\bar{Y}-Y(t-\theta));\ \beta>0,\ \bar{Y}\equiv\text{constant.}\geqq 0,\ \theta>0 \qquad (17)$$

where θ is the lag in policy response.

If we replace Eq. (5) in the previous section with Eq. (17), then, the fundamental dynamical equation becomes as follows.

$$\dot{Y}(t)=\alpha[I(Y(t),\ r(Y(t)))-\{1-c(1-\tau)\}Y(t)-\beta Y(t-\theta)+C_0+G_0+\beta\bar{Y}+cT_0]\equiv F(Y(t),\ Y(t-\theta);\ \beta) \qquad (18)$$

This is a simple type of the mixed difference and differential equation, the formal structure of which is somewhat similar to

Kalecki (1935)'s macrodynamic model of business cycle(5). The remaining part of this paper will be devoted to the analysis of the solution of this equation and the investigation of the economic implication of the solution.

III—1. Some Comparative Statics

Let us define the 'equilibrium national income' Y^* as the stationary solution of Eq. (18). Substituting $\dot{Y}(t)=0$ and $Y(t)=Y(t-\theta)=Y^*$ into Eq. (18), we have

$$h(Y^*)\equiv I(Y^*,\ r(Y^*))-\{1-c(1-\tau)+\beta\}Y^* +C_0+G_0+\beta\bar{Y}+cT_0=0. \tag{19}$$

Now, let us assume that

Assumption 2. $I(Y,\ r(Y))$ is bounded.

Under this assumption, we have

$$\lim_{Y\to+\infty} h(Y)=-\infty. \tag{20}$$

In addition, we have the following relationships.

$$h(0)=I(0,\ r(0))+(C_0+G_0+\beta\bar{Y}+cT_0) \tag{21}$$

$$h'(Y)=\underset{(+)}{I_Y}+\underset{(-)}{I_r}\ \underset{(+)}{r_Y}-\{1-c(1-\tau)+\beta\} \tag{22}$$

From these equations we have the following proposition.

Proposition 1.

If $I(0,r(0))>-(C_0+G_0+\beta\bar{Y}+cT_0)$ and $\beta>I_Y+I_r r_Y-\{1-c(1-\tau)\}$ for all $Y\geqq 0$, then, there exists the *unique* equilibrium national income $Y^*>0$ under *Assumption 2.*

Next, let us consider some comparative statics. If there exists an equilibrium national income, we obtain the following relationships solving Eq. (19) with respect to Y^*.

$$Y^*=Y^*(C_0,\ G_0,\ \bar{Y},\ T_0\ ;\ \beta)\ ;\ \partial Y^*/\partial C_0=\partial Y^*/\partial G_0=1/A,$$
$$\partial Y^*/\partial \bar{Y}=\beta/A,\ \ \partial Y^*/\partial T_0=c/A \tag{23}$$

where $A\equiv\beta-[I_Y^*+I_r^*\,r_Y^*-\{1-c(1-\tau)\}]\equiv\beta-a$.

Therefore, we have the following

Proposition 2.

Suppose that there exists an equilibrium national income $Y^*>0$. Then, we have $\partial Y^*/\partial C_0=\partial Y^*/\partial G_0\gtrless 0$, $\partial Y^*/\partial \bar{Y}\gtrless 0$ and $\partial Y^*/\partial T_0\gtrless 0$ according as $\beta\gtrless I_Y^*+I_r^*\,r_Y^*-\{1-c(1-\tau)\}\equiv a$.

In short, there exists the unique equilibrium national income $Y^*>0$ and the stational state multiplier $\partial Y^*/\partial G_0$ becomes positive if the government's fiscal parameter β is sufficiently large. On the other hand, if β is relatively small, $\partial Y^*/\partial G_0$ becomes negative. It will become clear later that the condition for positive multiplier ($\beta>a$) is also a *necessary* condition for the local stability of the system (see Eq. (27) (ii) in the next subsection).

III—2. Dynamic Stability of the System

Now, we are in a position to fully investigate the stability of the dynamical equation (18). Suppose that an equilibrium national income $Y^*>0$ exists. Then, the linear approximation of Eq. (18) around the equilibrium point can be expressed as

$$\dot{y}(t)=\alpha a\, y(t)-\alpha\beta\, y(t-\theta) \tag{24}$$

where $a\equiv[\partial\dot{Y}(t)/\partial Y(t)\Big|_{Y=Y^*}]\,/\,\alpha\equiv I_Y^*+I_r^*\,r_Y^*-\{1-c(1-$

$\tau)\}>0$, $y(t)\equiv Y(t)-Y^*$ and $y(t-\theta)\equiv Y(t-\theta)-Y^*$.
Substituting $y(t)=y(0)\,e^{\rho t}$ into Eq. (24) and rearranging, we have the following 'characteristic equation'.

$$\Gamma(\rho)\equiv\rho-\alpha a+\alpha\beta e^{-\theta\rho}=0 \tag{25}$$

or equivalently,

$$(1/\theta)\,\lambda-\alpha a+\alpha\beta e^{-\lambda}=0 \tag{25'}$$

where $\lambda\equiv\theta\rho$.

If all the roots of Eq. (25)′ have negative real parts, then the equilibrium point of the original dynamical system (18) is locally stable. On the other hand, if at least one root of Eq. (25)′ has positive real part, this system becomes locally unstable(6). It is worth to note that the characteristic equation (25)′ has infinite numbers of the complex roots besides the real roots the numbers of which are not more than two, and the complex solutions produce explosive or damping cyclical fluctuations according as the real part of the root is positive or negative(7). Hence, the full condition for the local stability requires that all roots including complex roots must have negative real parts. Next, let us fully investigate this condition for local stability. For this purpose, we can make use of the following mathematical theorem which is due to Hayes (1950).

Lemma 1. (Hayes' theorem)

All the roots of $H(\lambda)\equiv pe^{\lambda}+q-\lambda e^{\lambda}=0$, where p and q are real, have negative real parts if and only if

(i) $p<1$, and

(ii) $p<-q<\sqrt{(x^{*2}+p^2)}$,

where x^* is the root of $x=p\tan x$ such that $0<x<\pi$. If

$p=0$, we take $x^*=\pi/2$.

(Proof.)

See Hayes (1950) or Bellman and Cooke (1963) chap. 13.

(q. e. d.)

Now, we can rewrite the characteristic equation (25)′ as

$$H(\lambda)\equiv\theta\alpha a e^{\lambda}+(-\theta\alpha\beta)-\lambda e^{\lambda}\equiv p e^{\lambda}+q-\lambda e^{\lambda}=0 \quad (26)$$

where $p\equiv\theta\alpha a$ and $q\equiv-\theta\alpha\beta$. Therefore, we can write the full conditions for the local stability as follows in view of *Lemma 1*.

(i) $\theta<1/\alpha a$

(ii) $a<\beta$

(iii) $\beta<\surd\{(x^*/\theta\alpha)^2+a^2\}\equiv\phi(\theta)$, (27)

where x^* is the root of $g_1(x)\equiv(1/\theta\alpha a)x=\tan x\equiv g_2(x)$ such that $0<x<\pi$.

If the inequality (27) (i) is satisfied, we can express the solution of x^* graphically as in Fig. 1[8]. Moreover, we can see from this graph that $\tan x^*$ is a *decreasing* function of θ, i.e., $d(\tan x^*)/d\theta<0$, so that we have

$$d(x^*/\theta\alpha)/d\theta=a\, d(\tan x^*)/d\theta<0. \quad (28)$$

Hence, from (27)(iii) and Eq. (28) we have the following relationships.

(i) $\phi'(\theta)=[(x^*/\theta\alpha)\ d(x^*/\theta\alpha)/d\theta]\ /\ [\surd\{(x^*/\theta\alpha)^2+a^2]<0$

(ii) $\lim_{\theta\to 0}\phi(\theta)=\lim_{\theta\to 0}a\surd(\tan^2x^*+1)=+\infty$

(iii) $\lim_{\theta\to 1/\alpha a}\phi(\theta)=\lim_{\theta\to 1/\alpha a}a\surd(\tan^2x^*+1)=a$ [9] (29)

Now, let us define the 'stable domain' (S) as follows.

$$S\equiv\left\{(\beta,\ \theta)\epsilon R^2_{++}\ \middle|\ \text{All the roots of Eq. (26) have negative real parts}\right\}$$

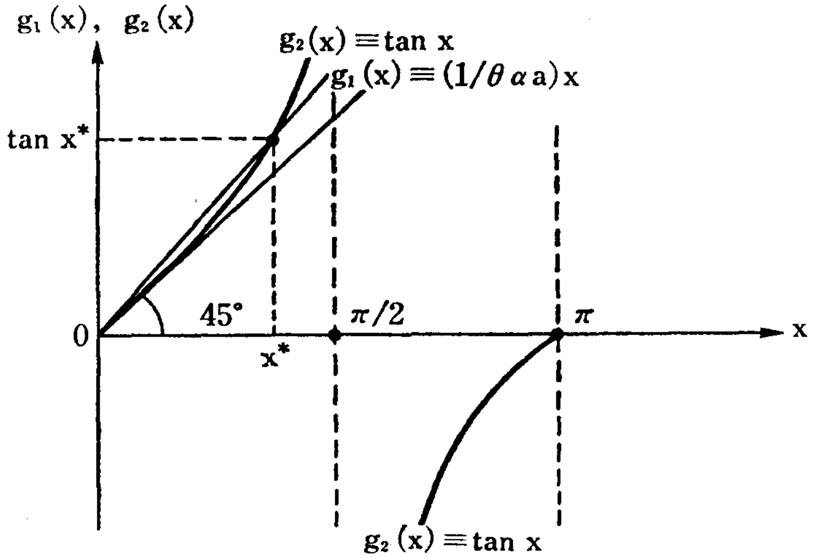

Fig. 1

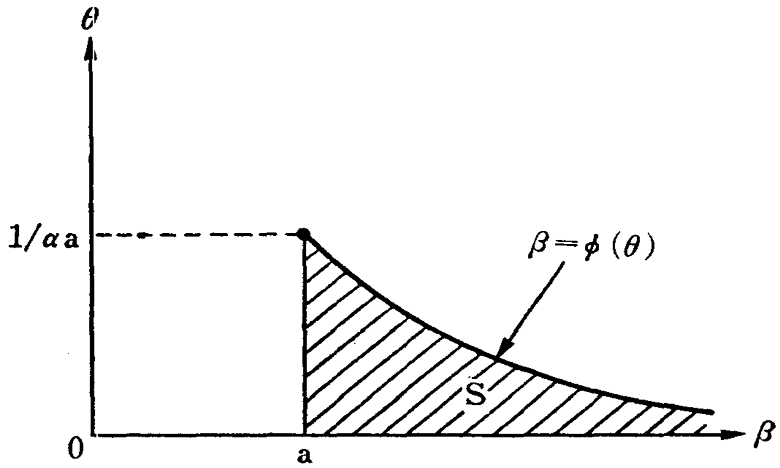

Fig. 2

$$\equiv\left\{(\beta,\ \theta)\epsilon R^2_{++}\ \middle|\ \theta < 1/\alpha a,\ \ a < \beta < \phi\ (\theta)\right\} \tag{30}$$

Then, we can illustrate the domain S as in Fig. 2 (boundary points are excluded).

The above analyses may be summarized as the following

Proposition 3.

(i) If $\theta > 1/\alpha a$, then, the equilibrium point of Eq. (18) is locally unstable irrespective of the value of $\beta > 0$.

(ii) If $0<\theta<1/\alpha a$, then, the equilibrium point of Eq. (18) is locally stable for $\beta\epsilon$ (a, $\phi(\theta)$) and it is locally unstable for $\beta\epsilon(0, a)\cup(\phi(\theta), +\infty)$, where $\phi(\theta)$ is a continuous *decreasing* function of θ and $\lim_{\theta\to 0}\phi(\theta)=+\infty$, $\lim_{\theta\to 1/\alpha a}\phi(\theta)=a$.

III—3. Comparative Dynamic Analysis

In this subsection, we shall analyze how the changes of some parameters affect the dynamic stability of the system by using the method of the comparative dynamic analysis.

Definition.

Let S_1 be the 'stable domain' S before the change of the parameter γ, and S_2 be the set S after the change of γ. Then, the change of γ is said to have 'stabilizing effect' if $S_1 \subsetneq S_2$, and it is said to have 'destabilizing effect' if $S_1 \supsetneq S_2$.

Proposition 4.

The increase of either of the parameter α and a has *destabilizing* effect.

(Proof.)

(i) It follows from Eq. (27) (iii) that $\phi(\theta)\equiv\sqrt{\{(x^*/\theta\alpha)^2+a^2\}}=\sqrt{\{(a\ \tan x^*)^2+a^2\}}=a\sqrt{(\tan^2 x^*+1)}=a\sqrt{\{d(\tan x)/dx\,|_{x=x^*}\}}$, while it is clear from Fig. 1 that $d(\tan x)/dx\,|_{x=x^*}$ is a *decreasing* function of α. Therefore, we have $\phi(\theta;\alpha_1)>\phi(\theta;\alpha_2)$ for all $\theta\epsilon$ $(0, 1/\alpha_2 a)$ if $\alpha_1<\alpha_2$. This

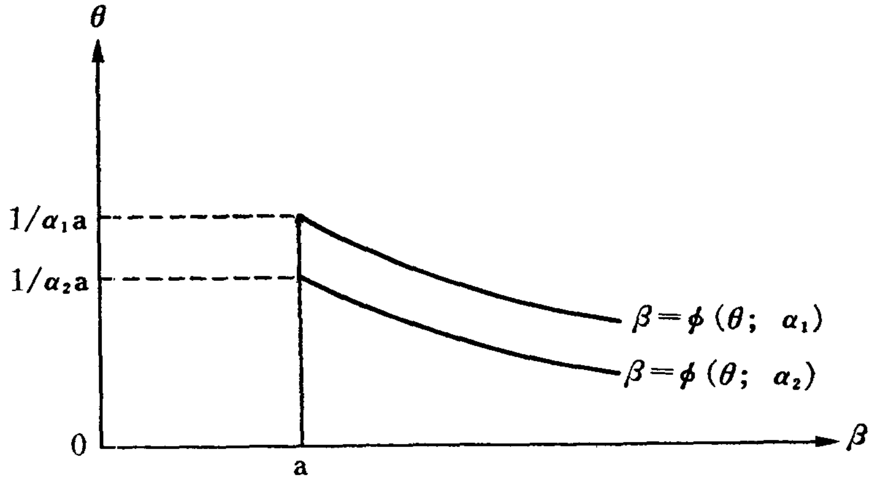

Fig. 3 $(0<\alpha_1<\alpha_2,\ s_1 \supsetneq s_2)$

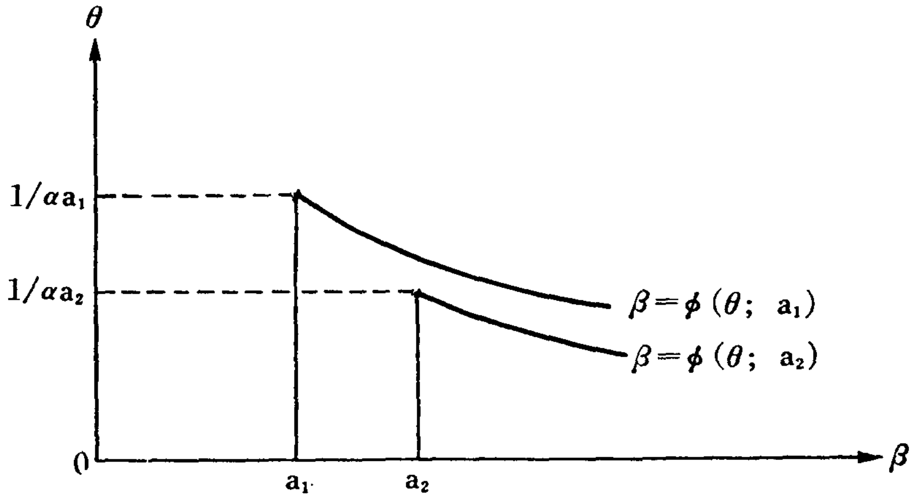

Fig. 4 $(0<a_1<a_2,\ s_1 \supsetneq s_2)$

implies that $S_1 \supsetneq S_2$ if $\alpha_1<\alpha_2$ (see Fig. 3).

(ii) It follows from Eq. (27) (iii) that $\phi(\theta)\equiv\sqrt{\{(x^*/\theta\alpha)^2+a^2\}}=\sqrt{\{(x^*/\theta\alpha)^2+(x^*/\theta\alpha\tan x^*)^2\}}=\sqrt{\{(x^*/\theta\alpha)^2(1+1/\tan^2 x^*)\}}$. On the other hand, we have $\partial\{(x^*/\theta\alpha)^2(1+1/\tan^2 x^*)\}/\partial x^*=\{2x^*(1+\tan^2 x^*)(\tan x^*-x^*)\}/\{(\theta\alpha)^2\tan^3 x^*\}>0$ for $x^*\epsilon(0,\pi/2)$[10]. Hence, we have $\partial\phi(\theta)/\partial x^*>0$ for $x^*\epsilon(0,\pi/2)$, from which it follows that $\partial\phi(\theta)/\partial a=(\partial\phi(\theta)/\partial x^*)(\partial x^*/\partial a)<0$ because we can see from Fig. 1 that x^* is a decreasing function of a. Therefore, we have $\phi(\theta\,;\,a_1)>\phi(\theta\,;\,a_2)$ for all $\theta\epsilon\,(0,\ 1/\alpha a_2)$ if $a_1<a_2$. This implies that $S_1 \underset{\neq}{\supset} S_2$ if $a_1<a_2$ (see Fig. 4).

(q. e. d.)

Corollary of Proposition 4.

(i) The increase of any of the parameters I_Y^*, $|L_r^*|$, m_r^* and c has *destabilizing* effect.

(ii) The increase of any of the parameters $|I_r^*|$, L_Y^*, p_Y^* and τ has *stabilizing* effect.

(Proof.)

From the definition of a, we have

$$a \equiv \underset{(+)}{I_Y^*}+\underset{(-)}{I_r^*}\,\underset{(+)}{r_Y^*}-\{1-c(1-\tau)\}$$

$$\equiv I_Y^*-|I_r^*|r_Y^*-\{1-c(1-\tau)\}$$

$$\equiv a(\underset{\oplus}{I_Y^*},\ \underset{\ominus}{|I_r^*|},\ \underset{\ominus}{r_Y^*},\ \underset{\oplus}{c},\ \underset{\ominus}{\tau}) \tag{31}$$

where $\underset{\oplus}{I_Y^*}$, $\underset{\ominus}{|I_r^*|}$ etc. mean that $\partial a/\partial I_Y^*>0$, $\partial a/\partial|I_r^*|<0$ etc.

On the other hand, it follows from Eq. (11) that

$$r_Y^*=-\{\underset{(+\text{ or }0)}{(m^*\bar{H}p_Y^*/p^{*2})}+\underset{(+)}{L_Y^*}\}/\{\underset{(-)}{L_r^*}-\underset{(+\text{ or }0)}{(m_r^*\bar{H}/p^*)}\}$$

$$=\{(m^*\bar{H}p_Y^*/p^{*2})+L_Y^*\}/\{|L_r^*|+(m_r^*\bar{H}/p^*)\}$$

$$=r_Y^*(\underset{\oplus}{L_Y^*},\ \underset{\oplus}{p_Y^*},\ \underset{\ominus}{|L_r^*|},\ \underset{\ominus}{m_r^*}). \tag{32}$$

Substituting Eq. (32) into Eq, (31), we obtain

$$a=\Phi(\underset{\oplus}{I_Y^*},\ \underset{\ominus}{|I_r^*|},\ \underset{\ominus}{L_Y^*},\ \underset{\ominus}{p_Y^*},\ \underset{\oplus}{|L_r^*|},\ \underset{\oplus}{m_r^*},\ \underset{\oplus}{c},\ \underset{\ominus}{\tau}). \tag{33}$$

Therefore, the increase of any of the parameters I_Y^*, $|L_r^*|$, m_r^* and c increases the parameter a, which has *destabilizing* effect. On the other hand, the increase of any of the parameters $|I_r^*|$, L_Y^*, p_Y^* and τ decreases the parameter a, which has *stabilizing* effect.

(q. e. d.)

Now, let us try to interprete the results of the analysis of this subsection by using the ordinary language of Macroeconomics.

Suppose the phase of prosperity with the upward trend of national income. The increase of the parameter I_Y^* or c accelerates this process because it induces further increase of the effective demand through the increase of the investment demand (the case of $\Delta I_Y^* > 0$) or the consumption demand (the case of $\Delta c > 0$). The increase of $|L_r^*|$ or m_r^* will also accelerate the above mentioned process because it will defend the rate of interest from rising. All of these factors have *destabilizing* effects on the macroeconomic process(11). The increase of the adjustment speed in the goods market (α) also has destabilizing effect because it speeds up the changes of national income.

On the other hand, the increase of $|I_r^*|$ or τ will suppress the process of prosperity through the negative effect on the effective demand. The increase of L_Y^* or p_Y^* will also suppress this process because it promotes to raise the rate of interest through the increase of the real demand for money (the case of $\Delta L_Y^* > 0$) or the decrease of the real money supply (the case of $\Delta p_Y^* > 0$). Obviously, these factors have *stabilizing* effect.

Ⅳ. Concluding Remarks

In this paper we have investigated the effect of the time lag in policy response on the (local) stability of the system in an analytical framework of the dynamic IS–LM model. In particular, we have shown that too long delay in policy response must fail to stabilize the system, and even if the policy lag is relatively short, too strong policy is not successful because of the 'overshooting' phenomena (see *Proposition 3*). Moreover, we have also shown that the existence of the policy lag will give rise to some cyclical movements apart from the business cycle intrinsic in the laissez faire capitalist economy. This cycle may be referred to as the 'policy cycle'.

In spite of these facts, it is *not* correct to say that the government's stabilization policy is entirely ineffective to stabilize the intrinsically unstable economy. In fact, if the policy lag is relatively short ($\theta < 1/\alpha a$), the government *can* stabilize the economic system by adopting a *positive* (but not too large) policy parameter β (see *Proposition 3*). In this sense, Keynesian (or Activistic) stabilization policy does not lose its significance even if we consider the lags in policy response explicitly(12).

Notes

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(1) Phillips (1954, 1957) reported the results of some numerical experimentations which are based on a theoretical model, but he did not investigate in detail the general relationships between the policy lag and

the dynamic stability of the system.

(2) This is virtually a simplified version of the model which was presented in Asada (1987). In Asada (1987), changes of the capital stock were explicitly introduced, but, in the present paper, we exclusively pay attention to Keynesian 'short run' for simplicity's sake, so that the changes of the capital stock are abstracted from. Similar model was also presented in Lorenz (1989).

(3) X(t) denotes the variable X at time t.

(4) In fact, this is the basic idea of the nonlinear theories of business cycle such as Kaldor (1940) or Goodwin (1981). See also Semmler (1986). Recently, Day (1989) showed the possibility of the chaotic behavior by using the discrete time version of the model which is similar to that in this paper.

(5) The use of the mixed difference and differential equation is somewhat unfamiliar to the standard economic analysis, but, we have some predecessors besides Kalecki (1935). See, for example, Steindl (1952), Johansen (1959) and Lange (1969).

(6) See Bellman and Cooke (1963) chap. 11.

(7) As for the proof, see Frisch and Holme(1935) or James and Belz(1938).

(8) Note that $d(\tan x)/dx \Big|_{x=0} = 1 + \tan^2 0 = 1$ and $1/\theta\alpha a > 1$ from the inequality (27) (i). In this case, there exists the unique solution $x^* \epsilon (0, \pi/2)$.

(9) Note that we have $\lim_{\theta \to 0} \tan x^* = +\infty$ and $\lim_{\theta \to 1/\alpha a} \tan x^* = 0$ from Fig. 1.

(10) Note that $\tan x^* > x^*$ at $x^* \epsilon (0, \pi/2)$ (see Fig. 1).

(11) We can argue quite symmetrically also in the case of the depression process.

(12) In this paper, the treatement of the financial market was kept as simple as possible to concentrate on the analysis of the policy lag. As for the more detailed analyses of the complicated dynamics of the financial market, see, for example, Minsiky (1986), Foley (1986), Taylor and O'Connel (1985) and Franke and Semmler (1989 a) (1989 b).

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